QUANTITATIVE FINANCE
RESEARCH CENTRE

UNIVERSITY OF
TECHNOLOGY SYDNEY

THINK.CHANGE.DO

QUANTITATIVE FINANCE RESEARCH CENTRE

Research Paper 364 September 2015

The Adaptiveness in Stock Markets: Testing the Stylized Facts in the Dax 30

Xue-Zhong He and Youwei Li

ISSN 1441-8010 www.qfrc.uts.edu.au

THE ADAPTIVENESS IN STOCK MARKETS: TESTING THE STYLIZED FACTS IN THE DAX 30

XUE-ZHONG HE* AND YOUWEI LI**

*University of Technology Sydney

Business School, Finance Discipline Group

PO Box 123 Broadway, NSW 2007, Australia

tony.he1@uts.edu.au

and

**Queen's University of Belfast

School of Management

Riddel Hall, 185 Stranmillis Road, BT9 5EE, Belfast, UK

y.li@qub.ac.uk

ABSTRACT. By testing a simple asset pricing model of heterogeneous agents to characterize the power-law behavior of the DAX 30 from 1975 to 2007, we provide supporting evidence on empirical findings that investors and fund managers use combinations of fixed and switching strategies based on fundamental and technical analysis when making investment decisions. By conducting econometric analysis via Monte Carlo simulations, we show that the autocorrelation patterns, the estimates of the power-law decay indices, (FI)GARCH parameters, and tail index of the model match closely the corresponding estimates for the DAX 30. A mechanism analysis based on the calibrated model provides further insights into the explanatory power of heterogeneous agent models.

Keywords: Adaptiveness, fundamental and technical analysis, stylized facts, power-law, tail index.

JEL Classification: C15, D84, G12

Date: Latest version: September 1, 2015.

Acknowledgements: This research was initiated and conducted during He's visit at Queen's University Belfast and Li's visit to Quantitative Finance Research Center at University of Technology Sydney, whose hospitality they gratefully acknowledge. Financial support from the Australian Research Council (ARC) under discovery grant (DP130103210) is also gratefully acknowledged. We benefited from detailed comments of anonymous referees, Michael Goldstein and Cars Hommes on the earlier version of this paper. The usual caveat applies.

*Corresponding author: Youwei Li, Queen's University of Belfast, School of Management, Riddel Hall, 185 Stranmillis Road, BT9 5EE, Belfast, UK. Email: y.li@qub.ac.uk. Ph: (44 28) 9097 4826. Fax: (44 28) 9097 4201.

1. Introduction

The use of fundamental and technical analysis by financial market professionals is well documented.[1] Empirical evidence suggests that investors and fund managers use combinations of fixed and switching strategies based on fundamental and technical analysis when making investment decisions. Recent laboratory experiments (e.g. Hommes *et al.*, 2005 and Anufriev and Hommes, 2012) provide further evidence on that agents use simple "rule of thumb" trading strategies and are able to coordinate on a common prediction rule, showing that heterogeneity in expectations is crucial to describe individual forecasting and aggregate price behavior. In this paper we test a simple asset pricing model of heterogeneous agents using the daily DAX 30 index[2] from 1975 to 2007. We show that the market is dominated by the adaptive investors who constantly switch between fundamental and trend following strategies, although some investors never change their strategies over the time. The results provide a strong support to the empirical evidence and laboratory experiments. Consequently, we provide further insights into the explanatory power of heterogeneous agent models to financial markets.

This paper is largely motivated by the recent literature on heterogeneity and bounded rationality. Due to limited information and endogenous uncertainty of the state of the world, investors are prevented from forming and solving life-time optimization problems in favor of more simple reasoning and rules of thumb (Shefrin, 2005). In general, investors are boundedly rational by making optimal decisions based on their limited information and expectations (Sargent, 1993). There is a growing evidence on investors' heterogeneity and bounded rationality, which has profound consequences for the interpretation of empirical evidence and the formulation of economic policy (Heckman, 2001). Research into asset pricing and financial market dynamics resulting from bounded rationality and interaction of adaptively heterogeneous traders has flourished over the last three decades and various heterogeneous agent models (HAMs) have been developed.[3] To explore the role of agents' heterogeneity in financial markets, the market

[1]See, for example, Allen and Taylor (1990), Taylor and Allen (1992), Menkhoff (1998) and Cheung *et al.*, (2004) for foreign exchange rate markets and Menkhoff (2010) for fund managers.

[2]The DAX, Deutscher Aktienindex (German stock index), tracks the segment of the largest and most important companies, known as blue chips, on the German equities market. It contains the shares of the 30 largest and most liquid companies admitted to the Frankfurt Stock Exchange in the Prime Standard segment. The DAX represents about 80% of the aggregate prime standards market cap. See *http://www.dax-indices.com/EN/index.aspx?pageID=1*.

[3]See, for example, Frankel and Froot (1990), Day and Huang (1990), Chiarella (1992), Lux (1995, 1998), Brock and Hommes (1998), Lux and Marchesi (1999), Hommes (2001), Chen and Yeh (2002), Farmer and Joshi (2002), Chiarella *et al.* (2002), Chiarella and He (2002, 2003), and De Grauwe and Grimaldi (2006).

dominance of different trading strategies represented by different types of traders plays a central role in market price behavior. It has been modelled either implicitly by examining their relative activity impacts, such as Day and Huang (1990) and Chiarella (1992) in early literature, or explicitly by examining their market fractions, such as Lux (1995), Brock and Hommes (1998), and Dieci, Foroni, Gardini and He (2006). The HAMs have successfully explained market booms, crashes, and deviations of the market price from the fundamental price. They are also able to replicate various stylized facts (including excess volatility, excess skewness, fat tails, volatility clustering and power-law behavior in return volatility) observed in financial markets.[4] The promising perspectives of the HAMs have motivated further empirical studies. Focusing on the model of Dieci et al. (2006), which allows for agents either having fixed strategies or switching their strategies based on past performance over time, we extend the model to include noise traders to rationalize the market noise in the model, our main contribution is then to systematically calibrate a large number of structural parameters of the model and subsequently perform series of formal econometric tests showing that the calibrated model is well able to replicate a large number of stylized facts.

This paper is closely related to a growing literature on the calibration and estimation of the HAMs in which the heterogeneity has been modeled through the well-known fundamentalists and chartists approach. These models have been successfully used to empirically explain speculation and bubble-like behavior in financial markets.[5] Despite the success such as Franke and Westerhoff (2011, 2012), econometric analysis and estimation of HAMs are still challenging tasks. The difficulties of estimation come from the complexity of the HAMs, together with (typically) many parameters, which makes verification of identification rather difficult, and thus proving consistency of estimation troublesome. Quite possibly a HAM might be misspecified, so that likelihood and/or moments based methods might produce poor results. But this situation is not alone when we look at literatures in other areas of economics and finance. In the real business cycles literature (Kydland and Prescott, 1982) and equity premium puzzle literature (Mehra

[4]We refer the reader to Hommes (2006), LeBaron (2006), Chiarella *et al.* (2009), Lux (2009*b*), and Chen *et al.* (2012) for surveys of recent developments in this literature.

[5]See, for instance, earlier works by Vigfusson (1997), Baak (1999), Chavas (2000), and for stock markets (Boswijk *et al.*, 2007; Franke, 2009; Franke and Westerhoff, 2011, 2012; Chiarella *et al.*, 2012, 2014; He and Li, 2015), foreign exchange markets (Westerhoff and Reitz, 2003; De Jong *et al.*, 2010; ter Ellen *et al.*, 2013), mutual funds (Goldbaum and Mizrach, 2008), option markets (Frijns *et al.*, 2010), oil markets (ter Ellen and Zwinkels, 2010), and sovereign European CDS spreads (Chiarella *et al.*, 2015). Also, HAMs have been estimated with contagious interpersonal communication by Gilli and Winker (2003), Alfarano *et al.* (2005), Lux (2009*a*, 2012), and other works reviewed in Li *et al.* (2010) and Chen *et al.* (2012).

and Prescott, 1985), we face with the very similar problems, and the models are assessed by calibration method. For example, in Kydland and Prescott (1982), the calibration consists in two steps. First, structural parameters are calibrated to values in previous empirical studies and to match long run average values. Second, the verification is implemented by judging the adequacy of the model to reproduce well chosen stylized facts, the parameters asides from those in the first step are treated as free parameters, their values are then chosen to minimize the distance between the well chosen stylized facts of the U.S. economy and the corresponding ones of the model. The calibration methodology is widely used in areas including Dynamic Stochastic General Equilibrium (DSGE) models. It does not consider the identification problem, precision of estimates and the goodness of fit are provided by the distance between the model and the data. It causes a huge amount of debate comparing with the usual estimation methodology where it attempts to find the parameters that lead to the best statistical fit by Maximum Likelihood (ML), Generalized Method of Moments (GMM), Method of Simulated Moments (MSM), or Efficient Method of Moments (EMM), and the performance of the model is examined through specification and goodness of fit tests.[6] The calibration and estimation are closest in spirit to Geweke (2006) classification of weak and strong econometric interpretation. The advantage of weak econometric interpretation is that the estimators are often more robust[7] than the full information estimators. In addition, it allows the researcher to focus on the characteristics in the data for which the model (which is necessarily an abstraction of reality) is most relevant. The attractions of strong econometric interpretation are clear, when successful, it provides a full characterisation of the data generating process and allows for proper specification testing. In existing works on estimation of HAMs, Franke (2009) applies MSM to a small model of Manzan and Westerhoff (2005) successfully. Franke and Westerhoff (2012) further develop model comparison method. The methods of Gilli and Winker (2003), Winker and Gilli (2003), Li *et al.* (2010) and He and Li (2015) belong to the weak econometric interpretation. HAMs are still in its infancy and they are very likely be misspecified. It is from this point we argue that this leaves room for weak econometric interpretation, it is an alternative to other existing ones,

[6]The debate are best summarized by Canova (1994), Hansen and Heckman (1996), Kydland and Prescott (1991, 1996), and Dridi *et al.* (2007).

[7]see, also in Diebold *et al.* (1998) and Schorfheide (2000).

and it is also usable for more complicated models. It can be interpreted in terms of consistent estimation of the parameter of interest (Dridi *et al.*, 2000).[8]

In this paper, following Li *et al.* (2010) and He and Li (2015) we take the weak econometric interpretation based on the power-law decay patterns of the autocorrelation of returns, the squared returns and the absolute returns for the DAX 30 stock market daily closing price index. We do this by choosing the interesting parameters in the whole model class that minimize the distance between particular actual data based autocorrelations and HAMs based autocorrelations. By conducting econometric analysis via Monte Carlo simulations, we show that the autocorrelation patterns, the estimates of the power-law decay indices, (FI)GARCH parameters, and tail index of the model match closely to the corresponding estimates for the DAX 30. Consequently, our results provide a strong support to the empirical evidence, including the popularity of fundamental and technical analysis, and boundedly rational and adaptive behavior of investors in financial markets.

The paper is structured as follows. Section 2 extends the adaptive asset pricing model developed in Dieci et al. (2006). Section 3 calibrates the model to characterize the power-law behavior of the DAX 30. Based on the calibrated parameters of the model, we use Monte Carlo simulations to examine the effectiveness of the calibration in generating the autocorrelation patterns, the decay indices of the power-law, and the tail behavior. Section 4 presents an explanation on the generating mechanism of the power-law behavior of the model. We also conduct formal tests to see how well the calibrated model is able to describe the characteristics of the DAX 30 and how the model fits better than a pure switching model. Section 5 concludes.

2. The Model

The use of technical analysis by financial market professionals is well documented. Empirical evidence (Allen and Taylor, 1990 and Taylor and Allen, 1992) suggests that the proportions of agents relying on particular strategies such as technical and fundamental analysis may vary over time, although there are certain *confident* agents who do not change their strategy over time. Recently, Menkhoff (2010) analyzes survey evidence from 692 fund managers in five

[8]The weak econometric interpretation is closely linked to the indirect inference methodology proposed by Gourieroux *et al.* (1993), which has been extended in Dridi *et al.* (2000, 2007). This methodology could gather both the advantages of the weak and strong econometric interpretation to consistently estimate some of the parameters of interests despite of model misspecification. Exploring on how to apply this methodology to HAMs might be a way forward in the future research.

countries. He finds that the share of fund managers that put at least some importance on technical analysis is very large. Though technical analysis does not dominate the decision-making of fund managers in general, but at a forecasting horizon of weeks, Menkhoff (2010) finds that technical analysis is the most important form of analysis and is thus more important than fundamental analysis, which is in line with findings from foreign exchange in Menkhoff (1998) and Cheung *et al.* (2004). Menkhoff (2010) strongly supports the view that heterogeneous agents possess different sets of information or different beliefs about market processes and the use of technical analysis seems to react to this view with trend-following behavior (and also by relying more strongly on momentum and contrarian investment strategies), believing that psychological factors are important and herding is beneficial. This view has also been shared by recent laboratory experiments in Hommes *et al.* (2005) and Anufriev and Hommes (2012). They show that agents using simple "rule of thumb" trading strategies are able to coordinate on a common prediction rule. Therefore heterogeneity in expectations is crucial to describe individual forecasting and aggregate price behavior.

Based on the empirical evidence, Dieci et al. (2006) extend early HAMs of Brock and Hommes (1998) by considering the case that market fractions have both fixed and adaptive switching components. In each trading period agents are assumed to be distributed among two groups, relying upon different predictors (or strategies, or behavioral rules), fundamental traders (or fundamentalists) and trend followers (or chartists). The market fractions in a given period are partially determined by the past performance of the strategies over time and partially fixed. In other words, a switching component is introduced to characterize *adaptively rational behavior* of agents who select different strategies over time according to a performance measure, and a constant component of agents is used to represent agents who are confident and stay with their strategies over time. While the fixed fraction component expresses the *market mood*, the switching fraction component captures the effect of *evolutionary adaption*. The focus of Dieci et al. (2006) is to explore the complicated price dynamics of the corresponding nonlinear deterministic model. Apart from the fundamentalists and trend followers, we also consider noise traders who play an important role in financial market (see, for example, Delong *et al.* 1990). In the following, we extend the model of Dieci et al. (2006) to include noise traders and show that the resulting model is actually the same as the model of Dieci et al. (2006).

Consider an asset pricing model with one risky asset and one risk free asset that is assumed to be perfectly elastically supplied at gross return $R = 1 + r/K$, where r is the constant risk free rate per annum and K is the frequency of trading period per year. Let p_t be the (ex dividend) price per share of the risky asset and $\{D_t\}$ the stochastic dividend process of the risky asset at time t. There are three types of traders (or investors/agents), fundamental traders (or fundamentalists), trend followers (or chartists) and noise traders, denoted by type $1, 2$ and 3 traders respectively. Let $Q_{i,t}(i = 1, 2, 3)$ be their market fractions at time t, respectively. We assume that there is a fixed fraction of noise traders, denoted by n_3. Among $1 - n_3$, the market fractions of the fundamentalists and trend followers have fixed and time varying components. Denote by n_1 and n_2 the fixed proportions of fundamentalists and trend followers among $1-n_3$, respectively. Then $(1 - n_3)(n_1 + n_2)$ represents the proportion of traders who stay with their strategies over time, while $(1 - n_3)[1 - (n_1 + n_2)]$ is the proportion of traders who may switch between the two types. Among the "switching" traders, we denote $n_{1,t}$ and $n_{2,t} = 1 - n_{1,t}$ the proportions of fundamentalists and trend followers at time t, respectively. It follows that the market fractions $(Q_{1,t}, Q_{2,t}, Q_{3,t})$ at time t are expressed by

$$Q_{1,t} = (1-n_3)[n_1+(1-n_1-n_2)n_{1,t}], \qquad Q_{2,t} = (1-n_3)[n_2+(1-n_1-n_2)n_{2,t}], \qquad Q_{3,t} = n_3.$$

Denote $n_0 = n_1 + n_2, m_0 = (n_1 - n_2)/n_0$ and $m_t = n_{1,t} - n_{2,t}$. Then the market fractions at time t can be rewritten as

$$\begin{cases} Q_{1,t} & = \frac{1}{2}(1 - n_3)\left[n_0\left(1 + m_0\right) + \left(1 - n_0\right)\left(1 + m_t\right)\right], \\ Q_{2,t} & = \frac{1}{2}(1 - n_3)\left[n_0\left(1 - m_0\right) + \left(1 - n_0\right)\left(1 - m_t\right)\right], \\ Q_{3,t} & = n_3 \end{cases} \tag{2.1}$$

Let $R_{t+1} := p_{t+1} + D_{t+1} - Rp_t$ be the excess return per share in $(t, t + 1)$. For $h = 1, 2$, let $E_{h,t}$ and $V_{h,t}$ be the conditional expectation and variance of type h traders. Let $W_{h,t}$ be investor's wealth at time t and $z_{h,t}$ the number of shares of the risky asset held by the investor from t to $t + 1$. Then the wealth of investor of type h at $t + 1$ is given by $W_{h,t+1} = RW_{h,t} + z_{h,t}(p_{t+1} + D_{t+1} - Rp_t)$. Assume that traders maximize the expected utility of wealth function $U_h(W) = -\exp(-a_h W)$, where a_h is the risk aversion coefficient of type h traders. Then, under the standard conditional normality assumption, the demand $z_{h,t}$ of a type h trader on the risky asset is given by $z_{h,t} = E_{h,t}(R_{t+1})/(a_h V_{h,t}(R_{t+1}))$.

Assume the demand of the noise traders is given by $\xi_t \sim N(0, \sigma_\xi^2)$, which is an i.i.d. random disturbance. With zero supply of outside shares, the population weighted average excess demand $Z_{e,t}$ at time t is given by

$$Z_{e,t} \equiv Q_{1,t}\, z_{1,t} + Q_{2,t}\, z_{2,t} + n_3 \xi_t.$$

Following Chiarella and He (2003), the market price in each trading period is determined by a market maker[9] who adjusts the price as a function of the excess demand. The market maker takes a long position when $Z_{e,t} < 0$ and a short position when $Z_{e,t} > 0$. The market price is adjusted according to

$$p_{t+1} = p_t + \lambda Z_{e,t}, \tag{2.2}$$

where λ denotes the speed of price adjustment of the market maker. Denote $\mu = (1 - n_3)\lambda$ and $\sigma_\delta = \lambda n_3 \sigma_\xi$. Then equation (2.2) becomes

$$p_{t+1} = p_t + \mu Z_{e,t} + \delta_t, \tag{2.3}$$

where $Z_{e,t} = q_{1,t}\, z_{1,t} + q_{2,t}\, z_{2,t}$ and $\delta_t \sim N(0, \sigma_\delta^2)$ with $q_{i,t} = Q_{i,t}/(1 - n_3)$ for $i = 1, 2$. The price equation (2.3) is exactly the model developed in Dieci et al. (2006).

We now describe briefly the heterogeneous beliefs of the fundamentalists and trend followers and the adaptive switching mechanism. This part is the same as in Dieci et al. (2006) and He and Li (2008). Fundamental traders are assumed to have some information on the fundamental value p^*_{t+1} of the risky asset at time t. They believe that the stock price may be driven away from the fundamental price in a short run, but it will eventually return to the fundamental value in a long-run. Thus the conditional mean and variance of the price for the fundamental traders are assumed to follow

$$E_{1,t}\,(p_{t+1}) = p_t + (1 - \alpha)(p^*_{t+1} - p_t), \qquad V_{1,t}\,(p_{t+1}) = \sigma_1^2, \tag{2.4}$$

where σ_1^2 is a constant variance on the price. The speed of adjustment towards the fundamental price is represented by $(1 - \alpha)$, where $0 < \alpha < 1$. An increase in α may thus indicate less confidence on the convergence to the fundamental price, leading to a slower adjustment.

[9]Different from the Walrasian equilibrium price mechanism used in Boswijk *et al.* (2007), we use market maker partial equilibrium mechanism for the convenience of calibration. The market maker mechanism has often been used in HAMs for its simplicity and convenience.

Unlike the fundamental traders, trend followers are assumed to extrapolate the latest observed price deviation from a long run sample mean price. More precisely, their conditional mean and variance are assumed to follow

$$E_{2,t}(p_{t+1}) = p_t + \gamma(p_t - u_t), \qquad V_{2,t}(p_{t+1}) = \sigma_1^2 + b_2 v_t, \tag{2.5}$$

where $\gamma \geq 0$ measures the extrapolation from the trend, u_t and v_t are sample mean and variance, respectively, which follow

$$u_t = \delta u_{t-1} + (1-\delta)p_t, \qquad v_t = \delta v_{t-1} + \delta(1-\delta)(p_t - u_{t-1})^2,$$

representing limiting processes of geometric decay processes when the memory lag tends to infinity.[10] Here $b_2 \geq 0$ measures the sensitivity to the sample variance and $\delta \in (0,1)$ measures the geometric decay rate. Note that a constant variance is assumed for the fundamentalists who believe the mean reverting of the market price to the fundamental price; while a time-varying component of the variance for the trend followers reflects the extra risk they take by chasing the trend.

We now specify how traders compute the conditional variance of the dividend D_{t+1} and of the excess return R_{t+1} over the trading period. For simplicity we assume that traders share homogeneous belief about the dividend process and that the trading period dividend D_t is i.i.d. and normally distributed with mean $\bar{D}$ and variance σ_D^2. The common estimate of the variance of the dividend (σ_D^2) is assumed proportional to the variance of the fundamental price, with no correlation between price and dividend. It follows that traders' conditional variances of the excess return can be estimated[11] as $V_{1,t}(R_{t+1}) = (1+r^2)\sigma_1^2$ and $V_{2,t}(R_{t+1}) = \sigma_1^2(1+r^2+bv_t)$, where $b = b_2/\sigma_1^2$. Denote by $p^* = \bar{D}/(R-1) = (K/r)\bar{D}$ the long-run fundamental price.

[10]With a geometric decaying probability distribution $(1-\delta)\{1,\delta,\delta^2,\delta^3,\cdots\}$ over the historical prices $\{p_t, p_{t-1}, p_{t-2}, p_{t-3}, \cdots,\}$, u_t and v_t are the corresponding sample mean and variance. See He (2003) for a detailed discussion on the process.

[11] The long-run fundamental value is given by $p^* = (K\bar{D})/r$, where $K\bar{D}$ is the average annual dividend. Let $\sigma\bar{p}$ be the annual volatility of the price p, where σ represents the annual volatility of 1 dollar invested in the risky asset. Under independent price increments, the trading period variance of the price can be estimated as $\sigma_1^2 = (p^*\sigma)^2/K$. Denote by D_A and $\sigma_{D_A}^2$ the annual dividend and its variance and assume an approximate relationship $D_A = rp$ between annual dividend and price. Then one gets $\sigma_{D_A}^2 = r^2(\sigma p^*)^2$ and therefore $\sigma_D^2 = \sigma_{D_A}^2/K = r^2(\sigma p^*)^2/K = r^2\sigma_1^2$. Assuming zero correlation between price and dividend at trading period frequency, one then obtain $V_{1,t}(R_{t+1}) = \left(1+r^2\right)\sigma_1^2$ and $V_{2,t}(R_{t+1}) = \sigma_1^2(1+r^2) + b_2 v_t$.

Using (2.4) and (2.5), it turns out that traders' optimal demands are determined by

$$z_{1,t}=\frac{(\alpha-1)\left(p_t-p^*_{t+1}\right)-(R-1)\left(p_t-p^*\right)}{a_1\left(1+r^2\right)\sigma_1^2},\quad z_{2,t}=\frac{\gamma\left(p_t-u_t\right)-(R-1)\left(p_t-p^*\right)}{a_2\sigma_1^2\left(1+r^2+bv_t\right)}. \tag{2.6}$$

Denote by $\pi_{h,t+1}$ the realized profit, or excess return, between t and $t+1$ by traders of type h, $\pi_{h,t+1}=z_{h,t}(p_{t+1}+D_{t+1}-Rp_t)=W_{h,t+1}-RW_{h,t}$ for $h=1,2$. Following Brock and Hommes (1997, 1998), the proportion of "switching" traders at time $t+1$ is determined by

$$n_{h,t+1}=\frac{\exp\left[\beta\left(\pi_{h,t+1}\right)\right]}{\sum_i\exp\left[\beta\left(\pi_{i,t+1}\right)\right]},\qquad h=1,2,$$

where parameter β is the *intensity of choice* measuring the switching sensitivity of the population of adaptively rational traders to the better profitable strategy. Together with (2.1) the market fractions and asset price dynamics are determined by the following random discrete-time dynamic system[12]

$$p_{t+1}=p_t+\mu(q_{1,t}\,z_{1,t}+q_{2,t}\,z_{2,t})+\delta_t,\qquad \delta_t\sim\mathcal{N}(0,\sigma_\delta^2), \tag{2.7}$$

$$u_t=\delta u_{t-1}+(1-\delta)\,p_t, \tag{2.8}$$

$$v_t=\delta v_{t-1}+\delta\,(1-\delta)\left(p_t-u_{t-1}\right)^2, \tag{2.9}$$

$$m_t=\tanh\left\{\frac{\beta}{2}\left(z_{1,t-1}-z_{2,t-1}\right)\left(p_t+D_t-Rp_{t-1}\right)\right\}, \tag{2.10}$$

$$D_t=\bar{D}+\sigma_D\nu_t,\qquad \nu_t\sim N(0,1), \tag{2.11}$$

where $z_{1,t}$ and $z_{2,t}$ are given by (2.6). The fundamental price is assumed to follow a random walk, such that[13]

$$p^*_{t+1}=p^*_t\exp(-\frac{\sigma_\epsilon^2}{2}+\sigma_\epsilon\epsilon_{t+1}),\qquad \epsilon_t\sim\mathcal{N}(0,1),\qquad \sigma_\epsilon\geq 0,\qquad p^*_0=p^*>0, \tag{2.12}$$

where ϵ_t is independent of the noisy demand process δ_t. The corresponding deterministic model can exhibit complicated price dynamics, which help us to understand the underlying mechanism

[12]Here the hyperbolic function $\tanh(x)$ is defined by $\tanh(x)=(e^x-e^{-x})/(e^x+e^{-x})$.

[13]The specification of the fundamental price process in (2.12) is to make sure that there is no significant ACs in returns, absolute returns and squared returns in the fundamental price. Since the focus of the paper is on the characteristics of returns, we also choose the fundamental price process p^*_t defined in equation (2.12) to have an expected mean value of zero. The long-run fundamental value $p^*=(K\bar{D})/r$ defined in Footnote 11 only indicates a reference long-run fundamental value, which is chosen as the initial value of the fundamental price process.

of the power-law behavior of the stochastic model. We refer the reader to Dieci *et al.* (2006) for the complex price dynamics and He and Li (2007) for a detailed discussion on the mechanism.

3. Estimation of the Power-Law behavior in the DAX 30

Econometric analysis, especially estimation, of HAMs is still a challenging task. In general, the difficulties of estimation come from the complexity of the HAMs, together with (typically) many parameters, which makes verification of identification rather difficult, and thus proving consistency of estimation troublesome, as we have discussed in the introduction. For recent attempts to estimate HAMs, the identification problem is typically circumvented by focussing on a relatively simple HAMs, or by estimating a few key parameters only. For example, Boswijk *et al.* (2007) derive a reduced and simplified Brock and Hommes (1997, 1998) type model and estimate it by using the nonlinear least square method; Alfarano *et al.* (2005) estimate a simplified herding model by the maximum likelihood method; Amilon (2008) estimates two specifications of the extended Brock and Hommes switching models by using the efficient method of moments and maximum likelihood method; Franke (2009) applies the method of simulated moments to a model developed by Manzan and Westerhoff (2005); Franke and Westerhoff (2012) use the same method to estimate a structural stochastic volatility HAM and show a strong herding component by conducting a model contest. Although a good progress seems to be made in estimating HAMs, even if consistent estimation was possible, the likely heavily nonlinear relationship between observables and unknown parameters to be estimated might seriously complicate estimation.[14]

This section provides a calibration of the model (2.7)-(2.12) to characterize the power-law behavior of the DAX 30. After a brief discussion of the stylized facts of the DAX 30, including both fat tail and power-law behavior, we introduce the calibration procedure to match the autocorrelation patterns in the returns, absolute and squared returns for the DAX 30, present the calibration result and conduct an out-of-sample test. Based on the calibrated parameters for the model, we use Monte Carlo simulations to examine the effectiveness of the calibration in generating the autocorrelation patterns and estimating the decay indices of the power-law behavior, comparing with those of the DAX 30. We also used the calibration result to examine the

[14] See, for example, Chen *et al.* (2012) and Amilon (2008). Amilon (2008) concludes that the simple prototype models seems to have potential to explain empirical facts although the fit is generally not quite satisfactory, he reports local minima, possibly not the global minimum, when calculating the estimators.

power-law tail behavior of the model comparing with the DAX 30. We show that the calibrated model closely generates the characterization of the power-law behavior of the DAX 30 in the return autocorrelation and tails.

3.1. **Stylized Facts and Autocorrelations of Returns for the DAX 30.** The price index data for the DAX 30 comes from Datastream, which contains 8001 daily observations from 11 August, 1975 to 29 June, 2007. We use p_t to denote the price index for the DAX 30 at time t $(t = 0, ..., 8000)$ with log returns r_t defined by $r_t = \ln p_t - \ln p_{t-1}$ $(t = 1, \cdots, 8000)$[15]. Table 3.1 gives the summary statistics of r_t for the DAX 30. We can see from Table 3.1 that the kurtosis for r_t is much higher than that of a normal distribution (which is 3). The kurtosis and studentized range statistics (which is the range divided by the standard deviation) show the characteristic fat-tailed behavior compared with a normal distribution. The Jarque-Bera normality test statistic is far beyond the critical value, which suggests that r_t is not normally distributed. Figures 3.1 (a) and (b) plot the time series of p_t and r_t, showing volatility clusterings and time-varying market volatility. This suggests that a suitable model for the data should be able to generate time varying volatility and volatility clustering as suggested by the ARCH and (FI)GARCH models.

TABLE 3.1. Summary statistics of r_t.

mean	std.	skewness	kurtosis	min	max	stud. range	Jarque-Bera
0.00034	0.01244	-0.4765	10.436	-0.1371	0.0755	17.092	18735

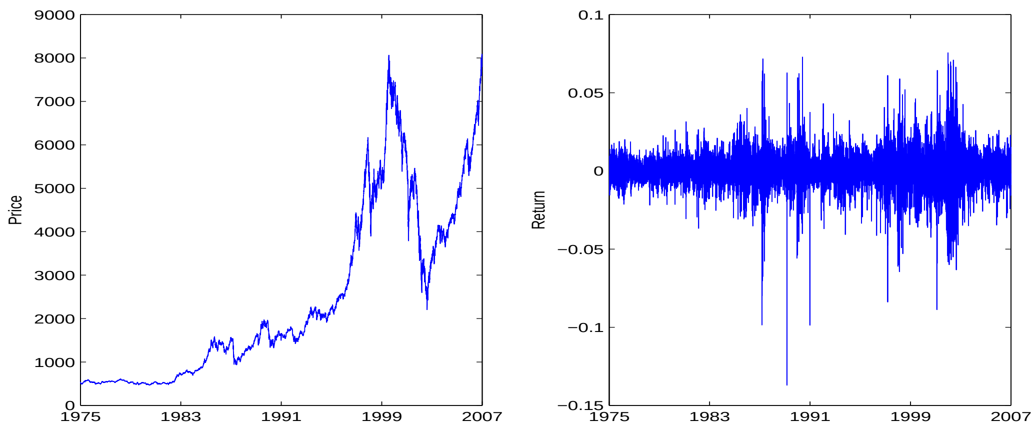


FIGURE 3.1. Time series on prices and log returns of the DAX 30 from 11 August, 1975 to 29 June, 2007.

[15]Note that at daily frequency, the difference between log-returns and simple returns is very small.

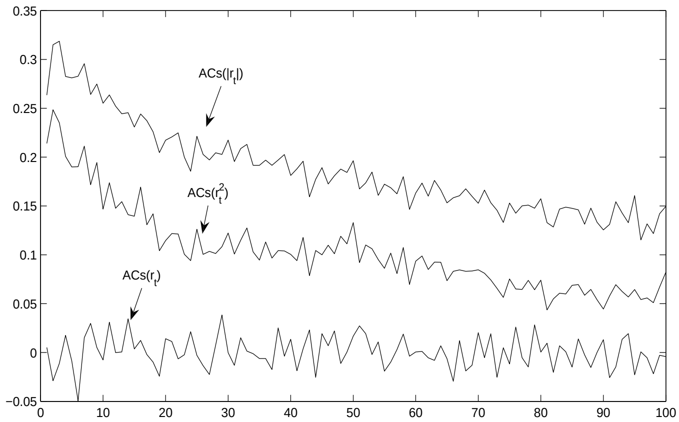


FIGURE 3.2. Autocorrelations of r_t, r_t^2 and $|r_t|$ for the DAX 30.

Among those reported stylized facts shared among different market indices, a well known stylized fact of stock returns is that the returns themselves contain little serial correlation, but the absolute returns $|r_t|$ and the squared returns r_t^2 do have significantly positive serial correlation over long lags. For example, Ding *et al.* (1993) investigate autocorrelations (ACs) of returns (and their transformations) of the daily S&P 500 index over the period 1928 to 1991 and find that the absolute returns and the squared returns tend to have very slow decaying autocorrelations, and further, the sample autocorrelations for the absolute returns are greater than those for the squared returns at every lag up to at least 100 lags. This kind of AC feature indicates the long-range dependence or the power-law behavior in volatility. The autocorrelations for the DAX 30 are plotted in Figure 3.2, which clearly support the findings in Ding *et al.* (1993).

3.2. **Model Calibration and Result.** In principle, to calibrate the power-law behavior of the DAX 30 to our model, we minimize the average distance between the autocorrelations of the log returns, the squared log returns, and the absolute log returns of the DAX 30 and the corresponding autocorrelations generated from the models[16]. More precisely, denote Θ the parameter space of the model. Let $\theta \in \Theta$ be the vector of parameters in the model to be calibrated, N be the number of independent simulations of the model, $\widehat{\beta}^n$ be the estimated autocorrelations of the n-th run of the model, and $\widehat{\beta}_{DAX}$ be that of the DAX 30. In calibration, we solve

$$\widehat{\theta} \in \arg\min_{\theta\in\Theta} D_\theta,\ D_\theta := \|\frac{1}{N}\sum_{n=1}^{N}\widehat{\beta}^n - \widehat{\beta}_{DAX}\|^2 \tag{3.1}$$

[16]Note that we do not consider other moments such as scales of returns and absolute returns and others. By exclusively focusing on the autocorrelations of return, squared return and absolute return, we provide a simple way to gain insight into the generating mechanism of power-law behavior of volatility of the model.

for the standard Euclidian norm $\|\cdot\|$, using an asynchronous parallel pattern search algorithm.[17] The parameters in the model are chosen to lie in the following ranges:[18] $\alpha \in [0,1]$, $\gamma \in [0.05, 5.5]$, $a_1, a_2 \in [0.001, 9.0]$, $\mu \in [0.1, 5]$, $m_0 \in [-1, 1]$, $n_0 \in [0.05, 0.995]$, $\delta \in [0, 1]$, $b \in [0.05, 8.5]$, $\beta \in [0.5, 1.5]$, $\sigma_\varepsilon \in [0.005, 0.05]$, $\sigma = \sqrt{K}\sigma_\varepsilon$ and $\sigma_\delta \in [0.05, 8.5]$. However $p_0^* = p^* = 100$, $q = r^2$, and $r = 0.05$ are kept fixed. In the calibration and the subsequent econometric analysis, we ran 1,000 independent simulations[19] over 9,000 time periods and discarded the first 1,000 time periods to wash out possible initial noise effect. For each run of the model we obtain 8,000 observations to match the sample size of the DAX 30. It is not possible to use autocorrelations at all lags, so we focus on a limited set of autocorrelations. In particular, we focus on all lags until 50 and then each fifth lag up to 100[20]. This corresponds to 60 autocorrelations in total for return, the absolute return and squared return, respectively. Essentially, with 60 autocorrelations estimated for each of the r_t, r_t^2 and $|r_t|$, the dimension of $\widehat{\beta}^n$ and $\widehat{\beta}_{DAX}$ is 180 in total. The calibrated parameters of the model are reported in Table 3.2.[21]

We note that HAMs are highly likely to be misspecified, the calibration procedure in (3.1) is based on the distance between the model and real world for a selected set of moments. It is designed to answer the question "given that the model is false, how true is it?" It allows us to focus on the characteristics in the data (in our case, this refers to the power law behavior in volatility) for which the model is most relevant. A related important question is "to find

[17]The software implementing the algorithm is APPSPACK 5.01, see more details in Gray and Kolda (2006), Griffin and Kolda (2006), and Kolda (2005). In the implementation, to avoid possible local minima we tried different set of starting values, and for each set of starting value we search for the minimum and then we re-initialize and search for the new minimum again. We repeat the procedure until there's no further improvements.

[18]The parameter ranges for α, m_0, n_o, δ are implied by the model specifications. The ranges for parameters γ, a_1, a_2 and μ are selected to reflect reasonable behavior of the traders based on the analysis of the underlying deterministic model in Dieci et al. (2006). The range for σ_ϵ represents the volatility of the fundamental price, while the range for σ_δ indicates the daily market price volatility level.

[19]Note that 1,000 simulation runs works well for us to produce accurate and relatively smooth ACs lines reported in Fig. 3.3, we do not consider the problem of the optimal number of simulations needed for solving this optimization problem. In other applications, much fewer number of simulation might be sufficient.

[20]We choose a large numbers of lags of ACs because our method of calibration of the model is exclusively focused on the ACs, and it works well to produce reasonable results reported in Fig. 3.3. In practice, much less lags may contain the same information and too many lags would waste computation time and even affect the accuracy of estimation, see for instance, Franke and Westerhoff (2012) for related discussion.

[21]It is likely that the estimated parameter values can be different for differ indices over different time periods. In fact, in our earlier exploratory model (He and Li, 2007, 2008, 2015 and Li *et al.*, 2010) using other indices or different periods of an index, the estimated model parameters are different in each of the cases. Quantitatively the stylized facts can vary over time, however, qualitatively the main feature of the stylized facts remains the same over long time periods and across different markets. It is this qualitative feature of the long memory pattern and the generating mechanism provided in Section 4.1 that this paper contributes to the current literature. It is from this perspective that the model estimation in this paper is robust. We would like to thank an anonymous referee for bringing up this discussion.

out how wrong a model is and to compare the performance of different models" (Kan and Robotti, 2008, 2009). In representative agent and rational expectation setting, measures of model misspecification developed by Hansen and Jagannathan (1997) and recently, Kan and Robotti (2008, 2009) are used to rank model performance. The distance in (3.1) is an analogue of Hansen and Jagannathan measure of model misspecification in the context of HAMs.[22]

TABLE 3.2. The calibrated parameters of the models

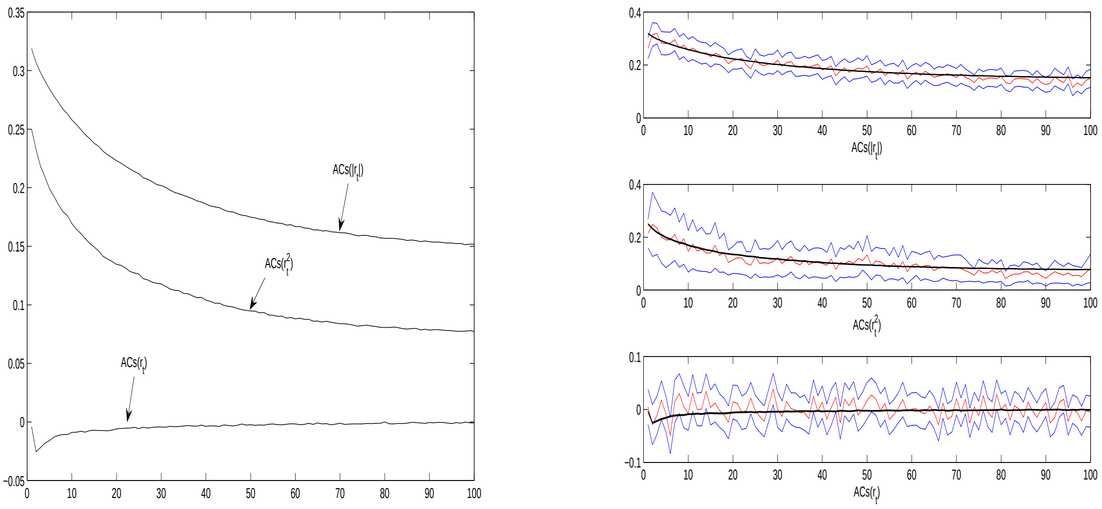
α	γ	a_1	a_2	μ	n_0	m_0	δ	b	σ	σ_δ	β
0.488	1.978	7.298	0.320	1.866	0.313	-0.024	0.983	3.537	0.231	3.205	0.954

We now provide an economic intuition of the calibrated result. Based on the calibrated parameters in Table 3.2, the parameter $n_o = 0.313$ implies that, among two strategies, there are some traders who do not change their investment strategies and most of traders switch between two strategies with a switching intensity measured by $\beta = 0.954$. This is consistent with the empirical evidence of using fundamental and technical analysis and the adaptive behavior of investors. With $m_o = -0.024$, it indicates that, among those traders who do not change their investment strategies, there are about equal numbers of trend followers and fundamentalists. These results demonstrate that both fundamentalists and trend followers are active in the market and the market is populated with confident traders as well as adaptive traders. This is in line with the findings from foreign exchange markets in Allen and Taylor (1990) and Taylor and Allen (1992) and fund managers in Menkhoff (2010). The relatively higher a_1 than a_2 implies that the fundamentalists are more risk averse than the trend followers[23]. A value of $\alpha = 0.488$ indicates that the speed of price adjustment of the fundamentalists towards the fundamental value is indicated by $1/(1-\alpha)$, which is about two trading periods. This may explain the frequent deviations of the market price from the fundamental value in short-run but not in long-run. A value of $\gamma = 1.978$ indicates that trend followers extrapolate the price trend, measured

[22]For HAMs, model comparison have been discussed in Li *et al.* (2010) and Franke and Westhoff (2012). Franke and Westhoff (2012) suggest measures of model comparison if the models can be successfully estimated by the methods of simulated moments. Developing measures using (approximated) stochastic discount factor would provide better insight into HAMs, however, this seems not feasible for the paper at the moment. Behavioral finance literature often finds limits of arbitrage (see, e.g., Shleifer and Vishny, 1997; Froot and Dabora, 1999; Lamont and Thaler, 2003; and Gromb and Vayanos, 2010), verification of existence of stochastic discount factor is not trivial, we plan to explore it further in future research.

[23]Note that for simplicity, we assume that agents' risk preferences switch when their strategies switch. Comparing to the trend followers who invest in short-run and are less risk averse, the fundamentalists invest in long-run and are more risk averse in general. We see from Footnote 11 that trend followers have a systematically higher variance estimate relative to the fundamentalists (by $bv_t\sigma_1^2$). When the additional term is much larger than $(1+r^2)\sigma_1^2$, the trend followers have much higher risk perception which also justifies the relative lower risk aversion of the trend followers than the fundamentalists.

by the difference between the current price and the geometric moving average of the history prices, actively. Also note that $\gamma = 1.978 > 1$ does not lead to explosive expectations by trend followers because of the quadratic volatility function in the denominator of the demand function. The geometric decay rate $\delta = 0.983$ indicates a slow decaying weight. The parameter $b_2 = b\sigma_1^2$ measures the influence of the sample variance v_t, in addition to the common belief on the price volatility σ_1^2, to the estimated price volatility for trend followers. The value of $b = 3.537$ implies that trend followers are cautious when estimating the price volatility, though they are less risk averse. The annual return volatility of $\sigma = 23.1\%$ is close to the annual return volatility of $19.67\%(= \sqrt{250} \times 0.01244)$ for the DAX 30. A value of $\mu = 1.866$ indicates that the market maker actively adjusts the market price to the excess demand of the traders. A positive σ_δ indicates that the noise traders are active in the market. In summary, the market is dominated by traders who switch between the two strategies based on their performance over the time, although there are some traders who do not change their strategies over the time. Due to the switching, the market becomes more volatile, which supports the theoretical predication in Brock and Hommes (1998), but in contrast to the finding in Amilon (2008) who find insignificant switching effect when estimating a structure HAM.



(a) (b)

FIGURE 3.3. (a) Autocorrelations of r_t, r_t^2 and $|r_t|$ for the model. (b) The ACs of the returns, the squared returns and the absolute returns for the calibrated model and the DAX 30. The smooth lines refer to the model while the 95% confidence intervals are those for the DAX 30.

3.3. **The Autocorrelation Patterns of the Calibrated Model and Out-of-Sample Test.** It is interesting to verify that our calibrated model is able to replicate the power-law behavior of the DAX 30 described in Fig. 3.2. Using the parameters in Table 3.2, we run 1,000 independent simulations for the model. For each run, we estimate the ACs for returns, squared returns and absolute returns. We then take the average over the 1,000 runs and plot the ACs in Fig. 3.3(a). It shows that for the model, the ACs are insignificant for the returns, but significantly positive over long lags for r_t^2 and $|r_t|$. Further, the sample autocorrelations for the absolute returns are greater than that for the squared returns at all lags up to at least 100 lags. Comparing with Fig. 3.2 for the DAX 30, we see that the patterns of decay of the autocorrelation functions of return, the squared return and the absolute return are very similar. To see how well the calibrated model is able to match the autocorrelations of r_t, r_t^2 and $|r_t|$ for the DAX 30, Fig. 3.3(b) plots the ACs of returns, the squared returns and the absolute returns for the model together with the DAX 30 respectively. For comparison purposes, we use the Newey-West corrected standard error and plot the corresponding 95% confidence intervals of the ACs of the DAX 30. It clearly indicates that all of the ACs of the model lie inside the confidence intervals of the DAX 30.[24]

We also perform an out-of-sample test for performance of the model,[25] Recall that we calibrate the model using the DAX 30 daily price index from 11 August 1975 to 29 June 2007, we now use data from 02 July 2007 to 02 April 2015 and plot ACs for returns, squared returns and absolute returns of the DAX 30 together with their 95% confidence intervals in Fig. 3.4, and to see if the ACs from the calibrated model fit in these intervals. We see from Fig. 3.4 that the ACs of returns and squared returns of the calibrated model fit in the 95% confidence intervals of the DAX 30 reasonably well, but the ACs of absolute returns of the calibrated model lie outside of the corresponding confidence intervals of the DAX 30 after lag 30, which indicate that the persistence in volatility of the DAX 30 is not as strong as before since the global financial crisis. Overall, the out-of-sample result indicates that the model performs reasonably well out of the sample and the calibration method effectively captures the ACs patterns of the DAX 30.

3.4. **Effectiveness of the Calibration.** Based on the calibrated parameters for the model, we use Monte Carlo simulations to further examine the effectiveness of the calibration in estimating the decay indices of the power-law behavior of ACs and in volatility clustering, comparing with

[24]Here we report the averages of the ACs based on 1,000 simulations and some of the ACs from a single simulation may lie outside the confidence band.

[25]We thank an anonymous referee for the suggestion.

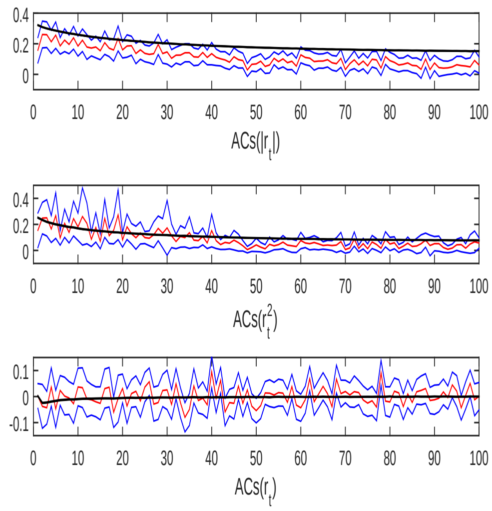


FIGURE 3.4. The ACs of the returns, the squared returns and the absolute returns for the calibrated model and the DAX 30. The smooth lines refer to the model while the 95% confidence intervals are those for the DAX 30 from 02 July 2007 to 02 April 2015.

those of the DAX 30. We also used the calibration result to examine the power-law tail behavior of the model comparing with the DAX 30. We show that the calibrated model closely generates the characterization of the power-law behavior of the DAX 30 in the return autocorrelation, volatility clustering and tails.

3.4.1. *Estimates of Power-law Decay Index.* Besides the visual inspection of ACs of r_t, r_t^2 and $|r_t|$, one can also construct models to estimate the decay rate of the ACs of r_t, r_t^2 and $|r_t|$. For instance, we can semiparametrically model long memory in a covariance stationary series x_t, $t = 0, \pm 1, ...$, by $s(\omega) \approx c_1\omega^{-2d}$ as $\omega \to 0^+$, where $0 < c_1 < \infty$, $s(\omega)$ is the spectral density of x_t, and ω is the frequency. Note that $s(\omega)$ has a pole at $\omega = 0$ for $0 < d < 1/2$ (when there is a long memory in x_t). For $d \geq 1/2$, the process is not covariance stationary. For $d = 0$, $s(\omega)$ is positive and finite. For $-1/2 < d < 0$, we have short memory, negative dependence, or antipersistence. The ACs can be described by $\rho_k \approx c_2k^{2d-1}$, where c_2 is a constant and $\mu \equiv 2d - 1$ corresponds to the hyperbolic decay index. In the literature, there are two most often used estimators of d, namely the Geweke and Poter-Hudak (1983), henceforth GPH, and Robinson and Henry (1999), henceforth RH. We describe the estimators and report the results in Appendix A.

For the DAX 30, we see from Table A.1 in Appendix A that the estimated d for the returns are not significant at any conventional significance levels but significant for the squared and the absolute returns. Thus the DAX 30 displays clear evidence of power-law for the squared and the absolute returns where d is positive, and the persistence in the absolute returns is much stronger than that in the squared returns. These results coincide with the well-established findings in the empirical finance literature. For the estimated model, the estimates of the decay rate d are reported in Table A.2 in Appendix A, where the column 'Sig%' indicates the percentage of simulations for which the corresponding estimates are significant at the 5% level over 1,000 independent simulations. We find that on average the estimates of d are negative and significantly different from those of the DAX 30 for returns at the 5% level (and insignificant at the 10% level), but significantly positive for the squared returns and the absolute returns. This verifies that there is a clear evidence of power-law for the squared returns and the absolute returns. It also shows that the patterns of the estimates of d for the squared returns and the absolute returns are comparable to those of the DAX 30 in Table A.1.

The above analysis clearly demonstrates that our calibration is effective in matching the autocorrelation patterns of the DAX 30. In the following discussion, we want to see if the calibrated model can be used to characterize the volatility clustering and power-law tail behavior, for which our calibration procedure is not designed.

3.4.2. *Volatility Clustering, Power-law and (FI)GARCH Estimates.* Another striking feature of the return series in market indices is *volatility clustering*. A number of econometric models of changing conditional variance have been developed to test and measure volatility clustering. The most widely used one is the one introduced by Engle (1982) and its generalization, the GARCH model, introduced by Bollerslev (1986). The GARCH implies that shocks to the conditional variance decay exponentially. In response to the finding that most of the financial time series are long memory volatility processes, Baillie *et al.* (1996) consider the Fractional Integrated GARCH (FIGARCH) process, where a shock to the conditional variance dies out at a slow hyperbolic rate. For convenience, in Appendix A, we describe the models and report the results.

Table A.3 in Appendix A reports the estimates of the GARCH $(1,1)$ model for the DAX 30, where the mean process follows an AR(1) structure. Based on estimates, one can see that a small influence of the most recent innovation (small α_1) is accompanied by a strong persistence of the variance coefficient (large β_1). It is also interesting to observe that the sum of the coefficients $\alpha_1 + \beta_1$ is close to one, which indicates that the process is close to an integrated GARCH (IGARCH) process. Such parameter estimates are rather common when considering returns from daily financial data of both stock and foreign exchange markets (see, Pagan (1996)). However the IGARCH implies that the shocks to the conditional variance persist indefinitely. Table A.4 in Appendix A reports the estimates of the FIGARCH $(1, \mathfrak{d}, 1)$ model for the DAX 30, where the mean process follows an AR(1) model. The estimate for the fractional differencing parameter $\hat{\mathfrak{d}}$ is statistically very different from both zero and one. This is consistent with the well known finding that the shocks to the conditional variance die out at a slow hyperbolic rate.

For the same specifications of the GARCH and FIGARCH models, we report resulting estimates for the calibrated model in Tables A.5 and A.6 in Appendix A, respectively. Again, all these estimates are the average of the estimations for each independent run of the calibrated model. The results from the GARCH model are very similar to that from the DAX 30, that is, a small influence of the most recent innovation is accompanied by strong persistence of the variance coefficient and the sum of the coefficients $\alpha_1 + \beta_1$ is close to one. For the estimates of the FIGARCH$(1, \mathfrak{d}, 1)$, we see that the estimate of $\mathfrak{d}$ for the calibrated model is significantly different from zero and one.

3.4.3. *Power-law Tail behavior.* Since the work of Mandelbrot (1963), power-law tail behavior has been found in a wide range of financial time series, and it has become one of the salient features in financial markets. In general, if f_{normal} is the probability density function of a normal distribution with mean μ and variance σ^2, then we have $\log f_{normal}(x) \sim -\frac{1}{2\sigma^2}x^2$ as $x \to \pm\infty$. A random variable X is said to follow a power-law or Pareto distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ if $Pr[X > x] = (x/\beta)^{-\alpha}$, for $x \geq \beta$. In this case, $\log f_{Pareto}(x) \sim -(\alpha + 1)\log(x)$ as $x \to +\infty$. Hence the difference of the tail behavior between the normal and Pareto distribution is significant.

The estimation of tail indices has been studied in great detail in extreme value theory. More precisely, let $X_1, X_2, ..., X_n$ be a sequence of observations from some distribution function F, with its order statistics $X_{1,n} \leq X_{2,n} \leq ... \leq X_{n,n}$. As an analogue to the central limit theorem, we know that, on average, if the maximum $X_{n,n}$, suitably centered and scaled, converges to a non-degenerate random variable, then there exist two sequences $\{a_n\}$ $(a_n > 0)$ and $\{b_n\}$ such that

$$\lim_{n\to\infty} \Pr\left(\frac{X_{n,n}-b_n}{a_n} \leq x\right) = G_\gamma(x), \tag{3.2}$$

where $G_\gamma(x) := \exp(-(1+\gamma x)^{-1/\gamma})$ for some $\gamma \in R$ and x such that $1+\gamma x > 0$. Note that for $\gamma = 0$, $-(1+\gamma x)^{-1/\gamma} = e^{-x}$. If (3.2) holds, then γ is called the extreme value index. In Pareto distribution, the tail index $\gamma := 1/\alpha$ measures the thickness of the tail distribution; the bigger the γ, the heavier the tail. The estimation of γ has been thoroughly studied, see Beirlant *et al.* (2004) for a detailed account. In Appendix A, we outline three major estimators of γ, the Hill estimator, the Pickands estimator, and the moment estimator in Dekkers *et al.* (1989) and report the corresponding results.

The Hill index relies on the average distance between extreme observations and the tail cutoff point to extrapolate the behavior of the tails into the broader part of the distribution. In practice, the behavior of the Hill index depends heavily on the choice of cutoff point k, which is also true for the other two estimators. This choice involves a tradeoff between bias and variance, which is well known in non-parametric econometrics. If k is chosen conservatively with few order statistics in the tail, then the tail estimate is sensitive to outliers in the distribution and has a high variance. On the other hand if the tail includes observations in the central part of the distribution, the variance is reduced but the estimate is biased upward. So, we plot these estimates index over a range of tail sizes. In the top panel of Fig. A.1 in Appendix A, we plot the Hill index. We see that for the negative tail, the Hill index of the model fits in the 95% confidence intervals of the DAX 30; for the positive tail, it fits well when k is less than 500. The Pickands estimates, plotted in the middle panel of Fig. A.1, show a larger variability. It seems that on average the estimates from the model are not far away from those of the DAX 30. The moment estimates, plotted in the bottom panel of Fig. A.1 for the model are slightly below the confidence intervals for the DAX 30. To conclude, the model exhibits power-law tail behavior which is very close to that of the DAX 30.

The overall analysis in this section shows that the calibration method is effective. The calibrated model is able to characterize successfully not only the power-law behavior in AC, but also the volatility clustering and power-law tail behavior in the DAX 30 as well.

4. Explanation and Comparison of the Calibration Results

We have shown that the calibrated model closely matches the stylized facts of the DAX 30. In this section, we provide an explanation on the generating mechanism of the power-law behavior of the model. In addition, we conduct formal tests to see how well the calibrated model is able to describe the characteristics of the DAX 30 and how the model fits better than a pure switching model.

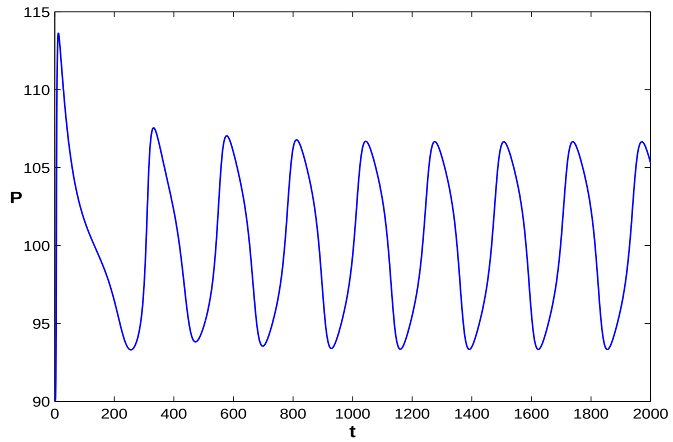
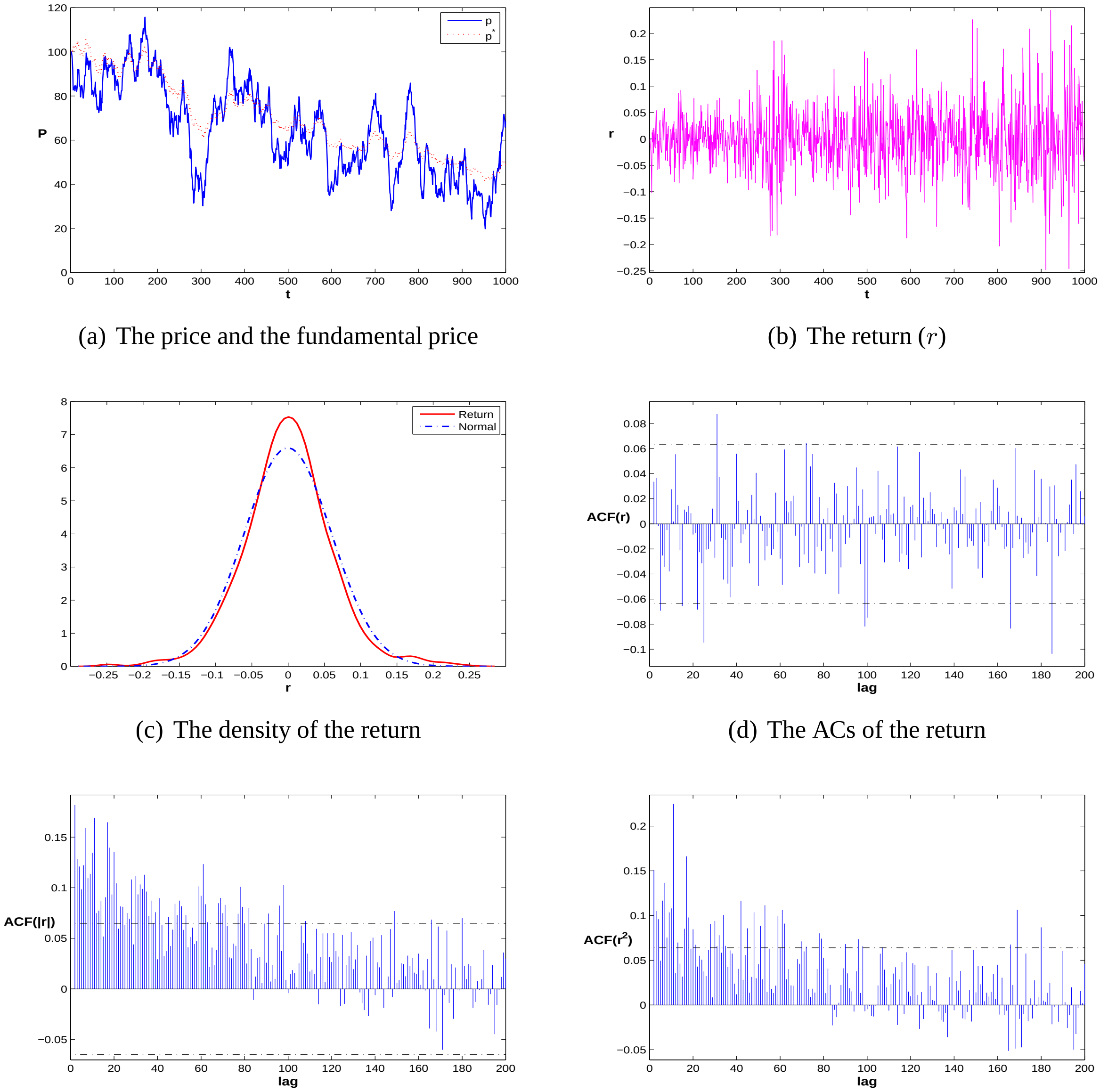


FIGURE 4.1. The price of the deterministic model with the calibrated parameters.

4.1. **Mechanism Analysis of the Power-Law Behavior.** With the help of the underlying deterministic dynamics, we now provide some insights into the mechanism of generating the power-law behavior. For the corresponding deterministic model with the calibrated parameters, the constant fundamental equilibrium becomes unstable,[26] leading to (a)periodical oscillation of the market price around the fundamental equilibrium, illustrated in Fig. 4.1. Such periodical deviations of the price from the fundamental value in the deterministic model are inherited in the stochastic model. Fig. 4.2(a) plots the time series of typical market price and fundamental price of the stochastic model. It shows that the price deviates from the fundamental price from

[26]The fundamental price becomes unstable through a so-called Hopf bifurcation. This is different from the mechanism provided in Gaunersdorfer *et al.* (2008) that volatility clustering is characterized by the underlying deterministic dynamics with two co-existing attractors with different sizes. In fact, the model developed in this paper can display such co-existence of locally stable fundamental price and periodic cycle, which has been demonstrated in Fig 3 in Dieci et al. (2006). Whether the model developed in this paper is able to provide a supporting evidence on the mechanism of Gaunersdorfer *et al.* (2008) would be an interesting issue for future research. We would like to thank Cars Hommes to bring our attention to this point.

time to time, but in general, follows the fundamental price. In addition, the returns of the stochastic model display the stylized facts of volatility clustering in Fig. 4.2(b) and non-normality of return distribution in Fig. 4.2(c).



(a) The price and the fundamental price

(b) The return (r)

(c) The density of the return

(d) The ACs of the return

(e) The ACs of the absolute return

(f) The ACs of the squared return

FIGURE 4.2. The time series of (a) the price (red solid line) and the fundamental price (blue dot line) and (b) the return; (c) the return density distribution; the ACs of (d) the returns; (e) the absolute returns, and (f) the squared returns.

The calibrated result provides a strong support on the power-law behavior mechanism reported in He and Li (2007). In He and Li (2007), a constant market fraction model is used to examine the potential source of agent-based models with heterogeneous belief in generating power-law behavior in return autocorrelation patterns. By examining the dynamics of the underlying deterministic model and simulating the impact of the fundamental noise and noise traders on the deterministic dynamics, He and Li (2007) find that the interaction of fundamentalists, risk-adjusted trend chasing from the trend followers and the interplay of noisy fundamental and demand processes and the underlying deterministic dynamics can be the source of power-law behavior. The calibrated model in this paper shares the same spirit of He and Li (2007). In fact, with the two noise processes, Fig. 4.2(d) demonstrates insignificant ACs for the returns, while Figs 4.2(e) and (f) show significant and decaying ACs in the absolute and squared returns, respectively. We also plot the times series of price, fundamental value, returns, return distribution, the ACs of return, absolute and squared returns with one noise, either the fundamental noise in Fig. C.1 or market noise in Fig. C.2, respectively, in Appendix C. They clearly demonstrate that, for the calibrated model, noise traders play an important role in the generation of insignificant ACs on the returns, while the significant decaying AC patterns of the absolute returns and squared returns are more influenced by the noisy fundamental process. This shows that the potential source of power-law generating mechanism obtained here shares the same spirit as He and Li (2007) and Chiarella, He and Hommes (2006).

4.2. **A Comparison Test.** To see how well the model is able to describe the characteristics in the DAX 30, we construct confidence intervals for the estimates based upon the DAX 30 to see if the estimates based upon the calibrated model lie in these intervals or not. In the following, we focus on the average estimates of the model rather than their accuracy since, by running the model independently many times, the estimates converge much faster than those of the DAX 30. Apart from checking the confidence intervals, we also construct the Wald test for this purpose. For instance, for the decay index d of the returns, the squared returns or the absolute returns, we test whether the values of the parameter d estimated from both the DAX 30 and the model are the same. In other words, we test hypothesis

$$H_0 : d_{DAX} = d.$$

Using the Wald test, this null hypothesis can be tested by assuming that both the number of simulations and the number of time periods for each simulation go to infinity. In the construction of the Wald test, the test statistic is given by

$$W = (\hat{d}_{DAX} - \hat{d})^2/\hat{\Sigma},$$

where $\hat{\Sigma}$ is simply the variance of $\hat{d}_{DAX}$[27]. The resulting test statistics are summarized in Table 4.1. In the column 'r_t', the first sub-row reports the test statistics corresponding to $\hat{d}_{GPH}$, and the second sub-row corresponding to $\hat{d}_{RH}$, and so on. Notice that the critical values of the Wald test at 5% and 1% significant levels are 3.842 and 6.635, respectively. For the returns, we see that the estimated d of the DAX 30 and the model are significantly different. However, for the squared returns and the absolute returns, the differences between the estimated d of the DAX 30 and the model are not statistically significant. This result shows that the calibrated model is able to describe the ACs of the absolute and squared returns in the DAX 30.

TABLE 4.1. The Wald test of d with $m = 50, 100, 150, 200, 250$

m	50	100	150	200	250
r_t	19.41	45.62	61.94	65.86	76.35
	35.41	92.24	126.0	117.5	129.4
r_t^2	0.071	1.309	0.282	0.036	0.023
	0.037	1.246	0.050	0.767	0.276
$\lvert r_t\rvert$	0.116	1.165	1.672	0.413	0.195
	0.020	0.350	0.067	0.031	0.015

Another comparison test is to see if the model (denoted SM) in Section 2 performs better than a pure switching model (denoted PSM) with $n_0 = 0$ in line of Brock and Hommes (1998). Intuitively, the calibration conducted for the SM should fit the data better than the PSM[28]. In Appendix B, we provide the calibrated parameters in Tab. B.1, the ACs patterns in Fig. B.1, the estimated decay indices in Tab. B.2, the GARCH and FIGARCH estimates in Tabs B.3 and B.4, the tail index plots in Fig. B.2, and the Wald test for the PSM. Apart from sharing similar results and implications to the SM, we calculate the distances of ACs, the $D_{\hat{\theta}}$ in Eq. (3.1), between the DAX 30 and the SM and PSM and obtain 4.56 and 4.59 respectively. The test statistics[29] $(\hat{\beta}_{DAX} - \hat{\beta})'\hat{\Omega}^{-1}(\hat{\beta}_{DAX} - \hat{\beta})$, where $\hat{\beta}$ is estimated from the simulation model and $\hat{\Omega}^{-1}$ is the generalized inverse (see, for example, Cameron and Trivedi, 2005) of corresponding covariance matrix, for ACs up to 50 lags for the return, the squared return and the absolute return of the SM and PSM are 106 and 108 respectively. Both results confirm that the SM performs better than

[27]We emphasize that the parameter uncertainty in $\hat{d}$ has not been taken into account because the simulations of the model are dependent on calibrated structural parameters.

[28]We notice that the main idea of this exercise is to show that the SM model can perform better than the PSW model in terms of generating stylized facts, which justifies the existence of agents in the market with fixed trading strategies in line with the model of He and Li (2007). So, we are not aiming to compare the SW model with various restricted version of the model to draw inference on the empirical importance of the SW model, we leave this to future research.

[29]The test statistics follows a Chi-square distribution with critical value 180 at the 5% significant level.

the PSM in terms of minimizing the distance in Eq. (3.1) and the weighted average distance by taking into account the $\hat{\Omega}$[30].

It is possible to develop measures of goodness of fit. While the measures of goodness of fit are very useful when comparing the performance of different HAMs (see, for example, Franke and Westerhoff, 2012), the comparison results on various econometric characterizations between HAM and the actual data seem to imply that it might be difficult to get meaningful test statistics. In our approach the sampling error from the actual data is dealt with the confidence intervals of the estimates and that from the simulation data is eliminated by running many independent simulation. For a more general discussion on the comparison of the simulation models with the real world data, see Li *et al.* (2006, 2010).

5. CONCLUSION

Theoretically oriented HAMs have provided many insights into market behavior such as market booming and crashing, multiple market equilibrium, short-run deviation of market price from the fundamental price and long-run convergence of the market price to the fundamental price. Combined with numerical simulations, the HAMs are able to reproduce some stylized fact, such as non-normality in return and volatility clustering. More recent developments in HAMs have stimulated many interests in the generation mechanism of those stylized facts and in particular, power-law behavior. However, estimation and calibration of HAMs to the power-law behavior of financial data, together with some mechanism explanation and economic intuition, are still a difficult and challenging task.

This paper calibrates an extended HAM to characterize the power-law behavior in the DAX 30. The model considers a market populated by heterogeneous traders who use either fundamental or chartist strategies. The market fractions of traders who use the two strategies have both fixed and switching components. The calibration method is based on minimization of the average distance between the autocorrelations (ACs) of the returns, the squared returns and the absolute returns of the DAX 30 and the corresponding ACs generated from the MF model. With the parameter values of the calibrated model, we show that the ACs of the market fraction model share the same pattern as the DAX 30. By conducting econometric analysis via Monte

[30]We emphasize that the comparison is based upon the magnitudes of distances we use. In other words, this is not to say that 4.56 (106) is significantly lower than 4.59 (108). A formal procedure such as that suggested by Hnatkovska *et al.*, (2012) might be explored further.

Carlo simulations, we estimate the decay indices, the (FI)GARCH parameters, Hill index and related tests. We show that the calibrated model matches closely to the corresponding estimates for the DAX 30. As a by-product, the calibrated model also generates non-normality return distribution, volatility clustering, and fat tails. Therefore the calibrated model can fit most of the stylized facts observed in the DAX 30.

The calibration results support the empirical evidence in financial markets that investors and fund managers use combinations of fixed and switching strategies based on various fundamental and technical analysis when making complicated investment decisions. By calibrating the model to the daily DAX 30 index from 1975 to 2007, we show that the market is dominated by the adaptive investors who constantly switch between the fundamental and trend following strategies, though there are some investors who never change their strategies over the time. In addition, the calibrated model also provides a consistent explanation on the generating mechanism of the power-law behavior in the literature. In conclusion, the calibration results provide strong support to the explanatory power of heterogeneous agent models and the empirical evidence of heterogeneity and bounded rationality.

APPENDIX A. ESTIMATES OF POWER-LAW DECAY INDEX, (FI)GARCH, AND POWER-LAW TAIL BEHAVIOR

This appendix provides the details of estimates of power-law decay index, (FI)GARCH, and power-law tail behavior.

A.1. **Power-law decay index.** Geweke and Poter-Hudak (1983) suggest a semiparametric estimator of the fractional differencing parameter d based on a regression of the ordinates of the log spectral density. Given spectral ordinates $\omega_j = 2\pi j/T$ $(j = 1, 2, ..., m)$, GPH suggest to estimate d from

$$\log I(\omega_j) = c - d\log(4\sin^2(\omega_j/2)) + v_j, \tag{A.1}$$

where v_j are assumed to be ***i.i.d.*** with zero mean and variance $\pi^2/6$. If the number of ordinates m is chosen such that $m = g(T)$ and satisfy $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty} g(T)/T = 0$ and $\lim_{T\to\infty}(\log(T)^2)/g(T) = 0$, then the OLS estimator of d based on (A.1) has the limiting distribution

$$\sqrt{m}(\hat{d}_{GPH} - d) \xrightarrow{d} \mathcal{N}(0, \frac{\pi^2}{24}). \tag{A.2}$$

Robinson (1995) provides a formal proof for $-1/2 < d < 1/2$, Velasco (1999) proves the consistency of $\hat{d}_{GPH}$ in the case $1/2 \leq d < 1$ and its asymptotic normality in the case $1/2 \leq d < 3/4$. It is clear from this result that the GPH estimator is not $\sqrt{T}$-consistent and in fact converges at a slower rate.

Another most often used estimator of d has been developed by Robinson and Henry (1999), they suggest a semiparametric Gaussian estimate of the memory parameter d, by considering

$$\hat{d}_{RH} = \arg\min_d R(d), \qquad R(d) = \log\left\{\frac{1}{m}\sum_{j=1}^{m}\omega_j^{2d} I(\omega_j)\right\} - 2\frac{d}{m}\sum_{j=1}^{m}\log\omega_j, \tag{A.3}$$

in which $m \in (0, [T/2])$. They prove that, under some conditions,

$$\sqrt{m}(\hat{d}_{RH} - d) \xrightarrow{d} \mathcal{N}(0, \frac{1}{4}) \tag{A.4}$$

when $m < [T/2]$ such that $1/m + m/T \to 0$ as $T \to \infty$.

A major issue in the application of the GPH and the RH estimators is the choice of m, due to the fact that there is limited knowledge available concerning this issue, see Geweke (1998) for instance. Hence it is a wise precaution to report the estimated results for a range of bandwidths. In our study, for both the GPH and the RH estimates of d, we report the corresponding estimates for $m = 50, 100, 150, 200$ and 250, respectively. For instance, for the DAX 30, Table A.1 reports the GPH and the RH estimates of d for returns, the squared returns, and the absolute returns, respectively. In each panel of Table A.1, the first row reports the results from the GPH and the RH estimates with $m = 50$, the second row reports the results of the GPH and the RH estimates with $m = 100$, and so on. Table A.2 is arranged similarly.

A.2. **(FI)GARCH.** Following the specification of Bollerslev (1986), if we model the returns as an AR(1) process, then a GARCH(p, q) model is defined by:

$$\begin{cases} r_t = a + br_{t-1} + \varepsilon_t, & \varepsilon_t = \sigma_t z_t, \\ \sigma_t^2 = \alpha_0 + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, & z_t \sim N(0,1), \end{cases} \tag{A.5}$$

where L is the lag operator, $\alpha(L) = \sum_{i=1}^{q}\alpha_i L^i$ and $\beta(L) = \sum_{j=1}^{p}\beta_j L^j$. Defining $v_t = \varepsilon_t^2 - \sigma_t^2$, the process can be rewritten as an ARMA(s, p) process

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \alpha_0 + [1 - \beta(L)]v_t \tag{A.6}$$

with $s = \max\{p, q\}$. Table A.3 reports the estimates of the GARCH $(1, 1)$ model for the DAX 30, where the mean process follows an AR(1) structure.

TABLE A.1. The estimates of d for the DAX 30 with $m = 50, 100, 150, 200, 250$

	$\hat{d}_{GPH}$	t	p-value	95% CI	$\hat{d}_{RH}$	t	p-value	95% CI
r_t	0.0014	0.014	0.989	[-0.2005, 0.2034]	-0.0179	-0.253	0.801	[-0.1565, 0.1207]
	0.0407	0.587	0.557	[-0.0954, 0.1769]	0.0615	1.229	0.219	[-0.0365, 0.1595]
	0.0548	0.985	0.325	[-0.0542, 0.1638]	0.0829	2.031	0.042	[0.0029, 0.1629]
	0.0406	0.852	0.394	[-0.0528, 0.1340]	0.0482	1.362	0.173	[-0.0211, 0.1175]
	0.0543	1.283	0.199	[-0.0286, 0.1372]	0.0571	1.807	0.071	[-0.0048, 0.1191]
r_t^2	0.4111	3.990	0.000	[0.2091, 0.6130]	0.3785	5.353	0.000	[0.2399, 0.5171]
	0.4527	6.518	0.000	[0.3165, 0.5888]	0.4365	8.731	0.000	[0.3385, 0.5345]
	0.4053	7.288	0.000	[0.2963, 0.5143]	0.3735	9.149	0.000	[0.2935, 0.4535]
	0.3666	7.696	0.000	[0.2733, 0.4600]	0.3508	9.923	0.000	[0.2816, 0.4201]
	0.3785	8.946	0.000	[0.2956, 0.4614]	0.3605	11.40	0.000	[0.2985, 0.4225]
$\|r_t\|$	0.5242	5.087	0.000	[0.3222, 0.7261]	0.4801	6.790	0.000	[0.3415, 0.6187]
	0.5495	7.911	0.000	[0.4133, 0.6856]	0.5167	10.33	0.000	[0.4187, 0.6147]
	0.5442	9.785	0.000	[0.4352, 0.6532]	0.4914	12.04	0.000	[0.4114, 0.5714]
	0.4993	10.48	0.000	[0.4059, 0.5927]	0.4818	13.63	0.000	[0.4125, 0.5511]
	0.4797	11.34	0.000	[0.3968, 0.5626]	0.4708	14.89	0.000	[0.4088, 0.5327]

TABLE A.2. The estimates of d for the model with $m = 50, 100, 150, 200, 250$

	$\hat{d}_{GPH}$	t	p-value	95% CI	Sig%	$\hat{d}_{RH}$	t	p-value	95% CI	Sig%
r_t	-0.4524	-4.390	0.060	[-0.4588, -0.4460]	83.9	-0.4386	-6.203	0.038	[-0.4430, -0.4342]	90.4
	-0.4287	-6.173	0.034	[-0.4330, -0.4244]	91.2	-0.4187	-8.374	0.026	[-0.4218, -0.4156]	94.0
	-0.3828	-6.883	0.030	[-0.3863, -0.3794]	92.6	-0.3750	-9.187	0.022	[-0.3776, -0.3725]	94.7
	-0.3457	-7.257	0.025	[-0.3487, -0.3428]	93.1	-0.3355	-9.488	0.016	[-0.3376, -0.3333]	95.3
	-0.3153	-7.453	0.024	[-0.3179, -0.3127]	93.1	-0.3023	-9.559	0.019	[-0.3043, -0.3003]	95.3
r_t^2	0.3836	3.723	0.022	[0.3772, 0.3900]	90.4	0.3920	5.544	0.002	[0.3876, 0.3964]	99.2
	0.3732	5.374	0.002	[0.3689, 0.3775]	99.2	0.3807	7.615	0.000	[0.3776, 0.3838]	100
	0.3758	6.758	0.000	[0.3724, 0.3793]	99.9	0.3826	9.372	0.000	[0.3801, 0.3851]	100
	0.3756	7.884	0.000	[0.3726, 0.3785]	100	0.3818	10.80	0.000	[0.3796, 0.3840]	100
	0.3721	8.795	0.000	[0.3695, 0.3747]	100	0.3771	11.92	0.000	[0.3751, 0.3790]	100
$\|r_t\|$	0.4891	4.747	0.003	[0.4827, 0.4954]	98.2	0.4902	6.932	0.000	[0.4858, 0.4946]	99.9
	0.4745	6.831	0.000	[0.4702, 0.4788]	100	0.4749	9.497	0.000	[0.4718, 0.4780]	100
	0.4723	8.493	0.000	[0.4689, 0.4758]	100	0.4731	11.59	0.000	[0.4706, 0.4757]	100
	0.4687	9.839	0.000	[0.4658, 0.4717]	100	0.4693	13.28	0.000	[0.4672, 0.4715]	100
	0.4610	10.90	0.000	[0.4584, 0.4637]	100	0.4621	14.61	0.000	[0.4601, 0.4640]	100

TABLE A.3. GARCH $(1, 1)$ Estimates for the DAX 30

$a \times 10^3$	b	$\alpha_0 \times 10^4$	α_1	β_1
0.4827	0.0539	0.0218	0.1056	0.8831
(0.1136)	(0.0127)	(0.0073)	(0.0232)	(0.0216)

Note: The numbers in parentheses are standard errors.

TABLE A.4. FIGARCH $(1, \mathfrak{d}, 1)$ Estimates for the DAX 30

a	b	$\alpha_0 \times 10^4$	$\mathfrak{d}$	ϕ_1	β_1
-0.0019	0.0012	0.0699	0.3259	0.2286	0.7716
(0.0003)	(0.0092)	(0.0248)	(0.0078)	(0.0148)	(0.0034)

Note: The numbers in parentheses are standard errors.

In response to the finding that most of the financial time series are long memory volatility processes, Baillie *et al.* (1996) consider the Fractional Integrated GARCH (FIGARCH) process, where a shock to the conditional variance dies out at a slow hyperbolic rate. Chung (1999) suggests a slightly different parameterization of the model:

$$\phi(L)(1-L)^{\mathfrak{d}}(\varepsilon_t^2-\sigma^2)=\alpha_0+[1-\beta(L)]v_t, \tag{A.7}$$

where $\phi(L)=1-\sum_{i=1}^{q}\phi_i L^i$, $\alpha_0=\phi(L)(1-L)^{\mathfrak{d}}\sigma^2$, and σ^2 is the unconditional variance of the corresponding GARCH model. Table A.4 reports the estimates of the FIGARCH $(1,\mathfrak{d},1)$ model for the DAX 30, where the mean process follows an AR(1) model. For the same specifications of the GARCH and FIGARCH models, we report resulting estimates for the estimated model in Tables A.5 and A.6, respectively.

TABLE A.5. GARCH $(1,1)$ Estimates for the Model

$a\times 10^3$	b	$\alpha_0\times 10^4$	α_1	β_1
-0.0754	0.0292	0.3084	0.0963	0.9084
(0.5974)	(0.0121)	(0.0886)	(0.0087)	(0.0076)
0.5	64.7	87.7	100	100

Note: The numbers in parentheses are the standard errors, and the numbers in the last row are the percentages that the test statistics are significant at 5% level over 1000 independent simulations. This also holds for Table A.6.

TABLE A.6. FIGARCH $(1,\mathfrak{d},1)$ Estimates for the Model

a	b	$\alpha_0\times 10^4$	$\mathfrak{d}$	ϕ_1	β_1
0.0325	0.0358	0.1314	0.4234	0.2108	0.7446
(0.0871)	(0.0296)	(0.1217)	(0.0642)	(0.0426)	(0.0413)
71.8	67.8	4.5	87.3	91.0	94.9

A.3. **Power-law tail behavior.** We outline three major estimators of γ, the Hill estimator, the Pickands estimator, and the moment estimator in Dekkers *et al.* (1989). The Hill index is defined by

$$H_{k,n}=\left(\frac{1}{k}\sum_{j=1}^{k}\log X_{n-j+1,n}\right)-\log X_{n-k,n}.$$

This estimator is consistent for $k\to\infty$, $k/n\to 0$ as $n\to\infty$, and under extra conditions, $\sqrt{k}(H_{k,n}-\gamma)$ is asymptotically normal with mean 0 and variance γ^2. The Pickands estimator is defined as

$$\hat{\gamma}_{P,k}=\frac{1}{\log 2}\log\left(\frac{X_{n-\lceil k/4\rceil+1,n}-X_{n-\lceil k/2\rceil+1,n}}{X_{n-\lceil k/2\rceil+1,n}-X_{n-k+1,n}}\right).$$

The simplicity of the Pickands estimator is appealing but offset by large asymptotic variance, equal to $\gamma^2(2^{2\gamma+1}+1)\{(2^\gamma-1)\log 2\}^{-2}$. Dekkers *et al.* (1989) introduce a moment estimator, which is a direct extension of Hill index,

$$M_{k,n}=H_{k,n}+1-\frac{1}{2}\left(1-\frac{H_{k,n}^2}{H_{k,n}^{(2)}}\right)^{-1},$$

where

$$H_{k,n}^{(2)}=\frac{1}{k}\sum_{j=1}^{k}(\log X_{n-j+1,n}-\log X_{n-k,n})^2.$$

They also prove the consistency and asymptotic normality. In Fig. A.1, we plot the estimates of the three tail estimators.

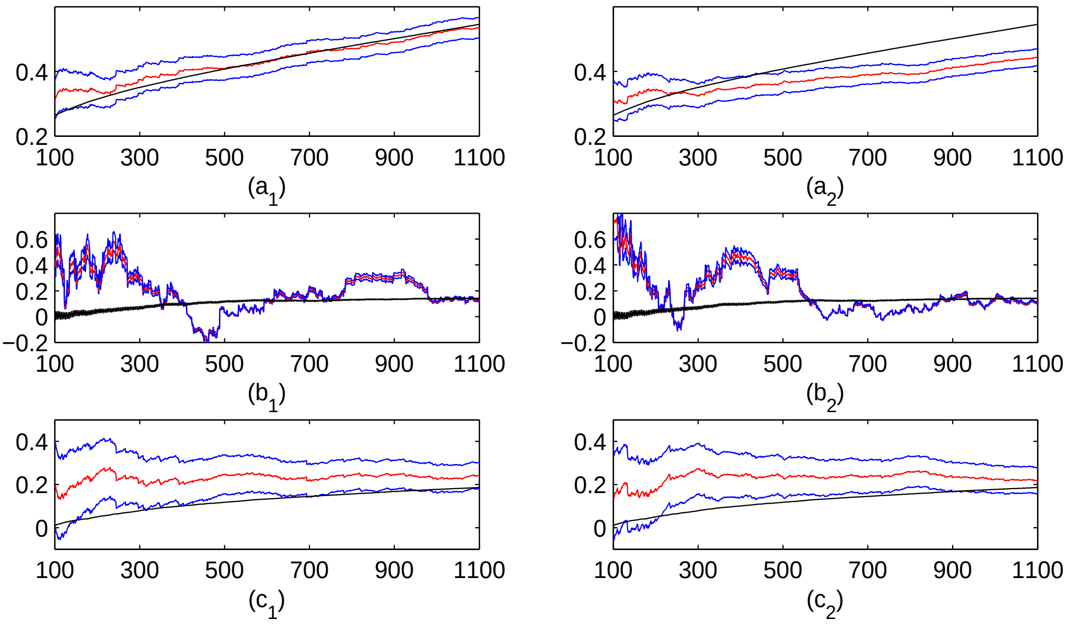


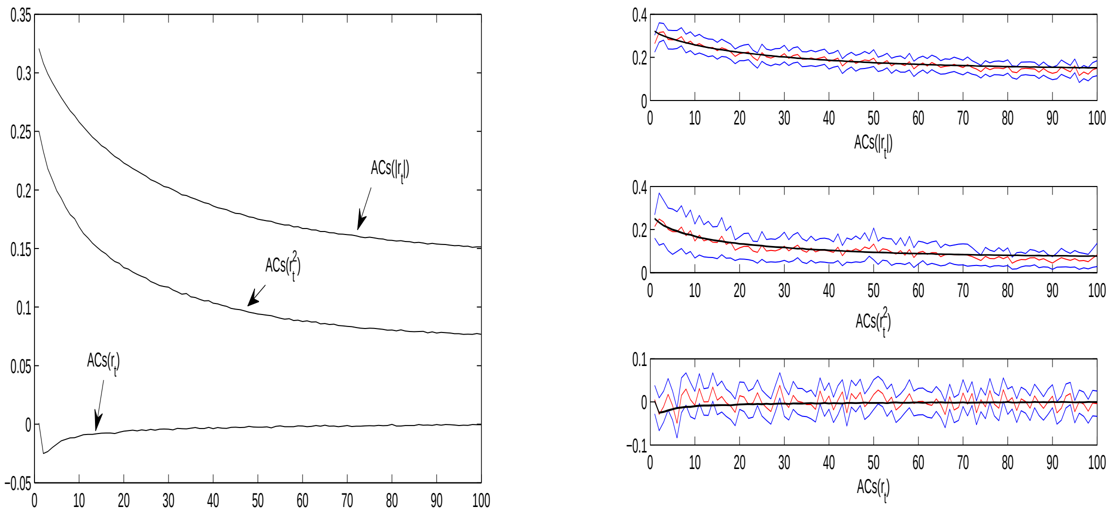
FIGURE A.1. The tail index plots $(k, H_{k,n})$, $(k, \hat{\gamma}_{P,k})$, and $(k, M_{k,n})$ of the negative tails (a_1), (b_1), (c_1) and the positive tails (a_2), (b_2), (c_2) for the SMF model and the DAX 30, respectively. The smooth lines refer to the model while the 95% confidence intervals are those for the actual data.

APPENDIX B. ECONOMETRIC ANALYSIS OF THE PURE SWITCHING MODEL

This Appendix provides calibration results of the pure switching model (2.7)-(2.12) with $n_o = 0$ to characterize the power-law behavior of the DAX 30.

TABLE B.1. The calibrated parameters of the SW models

α	γ	a_1	a_2	μ	δ	b	σ	σ_δ	β
0.513	0.764	7.972	0.231	2.004	0.983	3.692	0.231	3.268	0.745



(a) (b)

FIGURE B.1. (a) Autocorrelations of r_t, r_t^2 and $|r_t|$ for the SW model. (b) The ACs of the returns, the squared returns and the absolute returns for the calibrated SW model and the DAX 30. The smooth lines refer to the SW model while the 95% confidence intervals are those for the DAX 30.

TABLE B.2. The estimates of d for the SW model with $m = 50, 100, 150, 200, 250$

	$\hat{d}_{GPH}$	t	p-value	95% CI	Sig%	$\hat{d}_{RH}$	t	p-value	95% CI	Sig%
r_t	-0.4466	-4.334	0.059	[-0.4530, -0.4402]	84.1	-0.4361	-6.168	0.037	[-0.4405, -0.4318]	90.6
	-0.4241	-6.106	0.033	[-0.4284, -0.4198]	91.9	-0.4159	-8.317	0.025	[-0.4190, -0.4128]	94.2
	-0.3816	-6.861	0.028	[-0.3850, -0.3781]	92.6	-0.3746	-9.175	0.021	[-0.3771, -0.3720]	95.1
	-0.3466	-7.277	0.024	[-0.3496, -0.3437]	94.1	-0.3373	-9.539	0.015	[-0.3395, -0.3351]	96.1
	-0.3176	-7.506	0.023	[-0.3202, -0.3149]	94.2	-0.3059	-9.673	0.016	[-0.3078, -0.3039]	95.6
r_t^2	0.3843	3.730	0.021	[0.3779, 0.3907]	90.1	0.3918	5.540	0.002	[0.3874, 0.3961]	99.2
	0.3751	5.400	0.001	[0.3708, 0.3794]	99.6	0.3801	7.603	0.000	[0.3770, 0.3832]	100
	0.3768	6.776	0.000	[0.3734, 0.3803]	99.9	0.3815	9.345	0.000	[0.3790, 0.3840]	100
	0.3754	7.879	0.000	[0.3724, 0.3783]	100	0.3803	10.76	0.000	[0.3781, 0.3825]	100
	0.3717	8.786	0.000	[0.3691, 0.3743]	100	0.3758	11.88	0.000	[0.3738, 0.3778]	100
$\|r_t\|$	0.4909	4.765	0.003	[0.4845, 0.4973]	98.6	0.4910	6.943	0.000	[0.4866, 0.4954]	100
	0.4771	6.869	0.000	[0.4728, 0.4814]	100	0.4760	9.520	0.000	[0.4729, 0.4791]	100
	0.4738	8.519	0.000	[0.4703, 0.4772]	100	0.4735	11.60	0.000	[0.4710, 0.4761]	100
	0.4687	9.839	0.000	[0.4658, 0.4717]	100	0.4693	13.27	0.000	[0.4671, 0.4715]	100
	0.4609	10.89	0.000	[0.4583, 0.4636]	100	0.4618	14.60	0.000	[0.4598, 0.4637]	100

TABLE B.3. GARCH $(1,1)$ Estimates for the SW Model

$a \times 10^3$	b	$\alpha_0 \times 10^4$	α_1	β_1
-0.0660	0.0351	0.3141	0.0971	0.9078
(0.6081)	(0.0121)	(0.0905)	(0.0089)	(0.0078)
0.5	68.2	87.8	99.8	100

Note: The numbers in parentheses are the standard errors, and the numbers in the last row are the percentages that the test statistics are significant at 5% level over 1000 independent simulations. This also holds for Table B.4.

TABLE B.4. FIGARCH $(1,\mathfrak{d},1)$ Estimates for the SW Model

a	b	$\alpha_0 \times 10^4$	$\mathfrak{d}$	ϕ_1	β_1
0.0410	0.0244	0.1229	0.4282	0.1981	0.7578
(0.2272)	(0.0694)	(0.1311)	(0.0899)	(0.1519)	(0.0578)
72.6	66.2	4.2	88.3	90.7	96.1

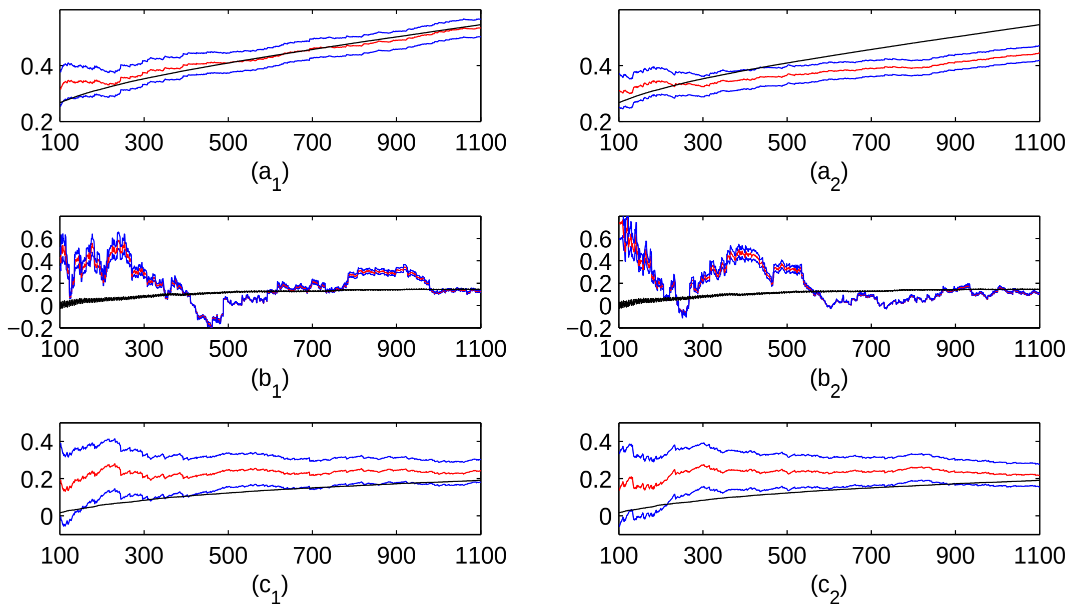


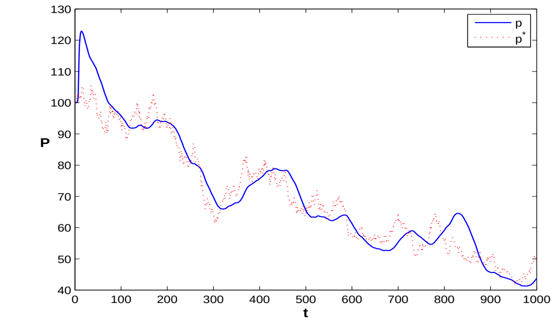
FIGURE B.2. The tail index plots $(k, H_{k,n})$, $(k, \hat{\gamma}_{P,k})$, and $(k, M_{k,n})$ of the negative tails (a_1), (b_1), (c_1) and the positive tails (a_2), (b_2), (c_2) for the SW model and the DAX 30, respectively. The smooth lines refer to the SW model while the 95% confidence intervals are those for the actual data.

TABLE B.5. The Wald test of d with $m = 50, 100, 150, 200, 250$

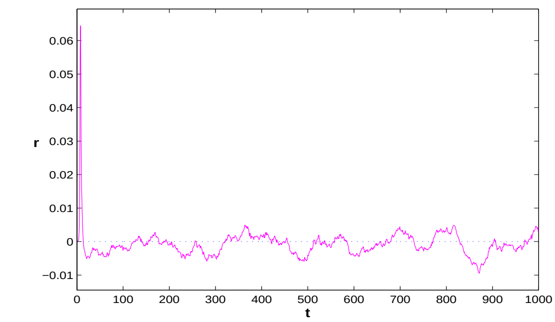
m	50	100	150	200	250
r_t	18.92	44.73	61.61	66.17	77.30
	34.99	91.16	125.7	118.6	132.0
r_t^2	0.068	1.247	0.263	0.034	0.026
	0.035	1.272	0.038	0.694	0.234
$\lvert r_t \rvert$	0.105	1.085	1.603	0.413	0.198
	0.024	0.331	0.064	0.031	0.016

Appendix C. The Effect of One Noise

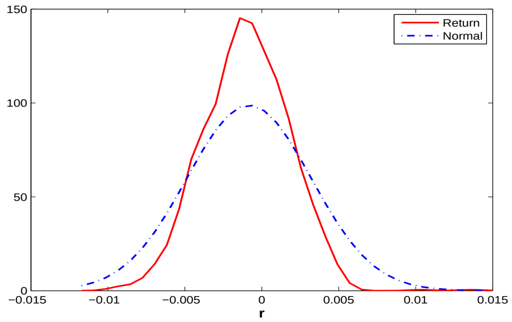
This appendix demonstrates the impact of single noise in the model (2.7)-(2.12) on the AC patterns of the return, absolute returns and squared returns.



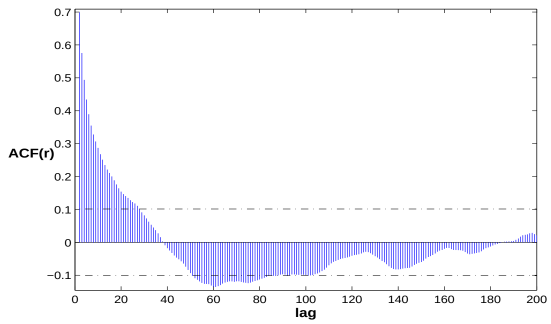
(a) The price and the fundamental price



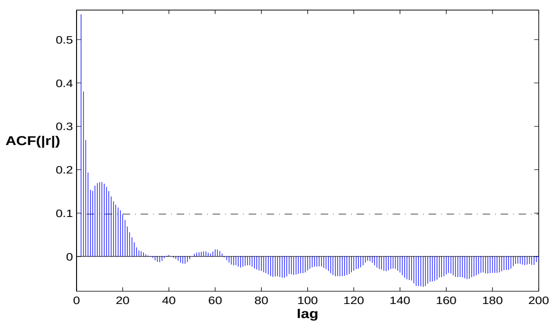
(b) The return (r)



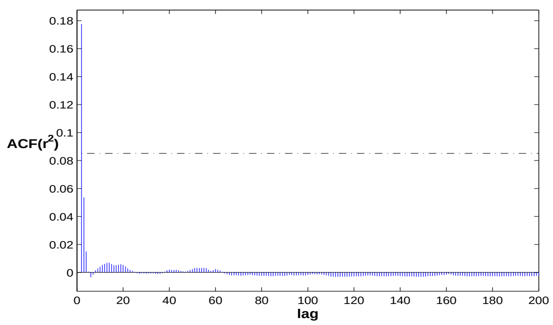
(c) The density of the return



(d) The ACs of the return

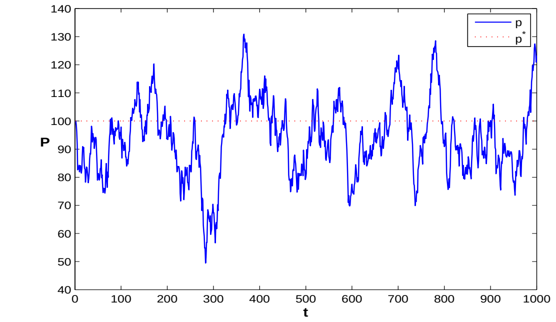


(e) The ACs of the absolute return

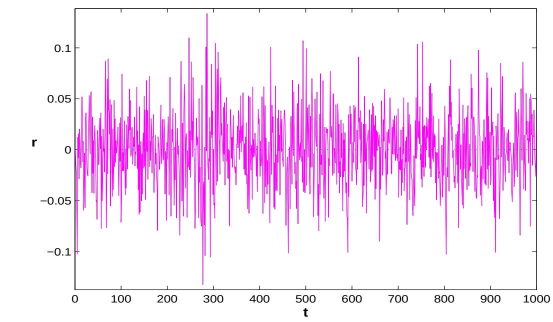


(f) The ACs of the squared return

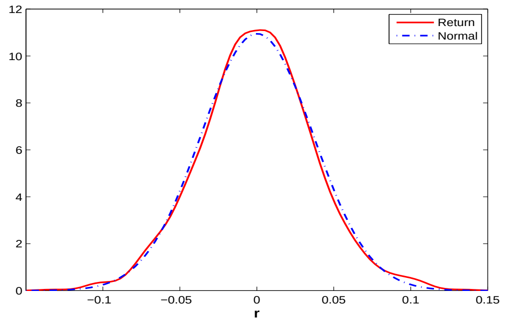
FIGURE C.1. The time series of (a) the price (red solid line) and the fundamental price (blue dot line) and (b) the return; (c) the density distribution of the returns; the ACs of (d) the returns; (e) the absolute returns, and (f) the squared returns, with the fundamental noise only ($\sigma_\delta = 0$).



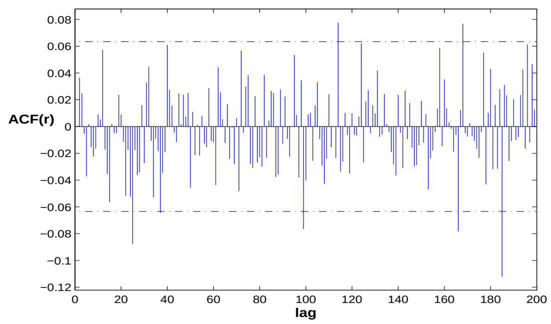
(a) The price and the fundamental price



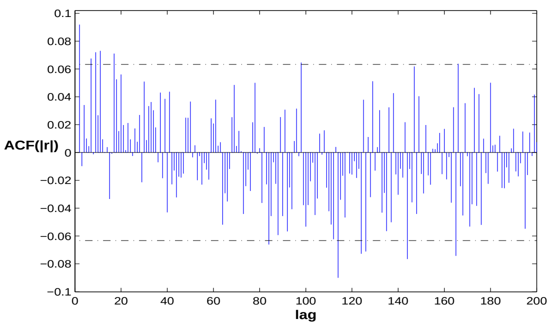
(b) The return (r)



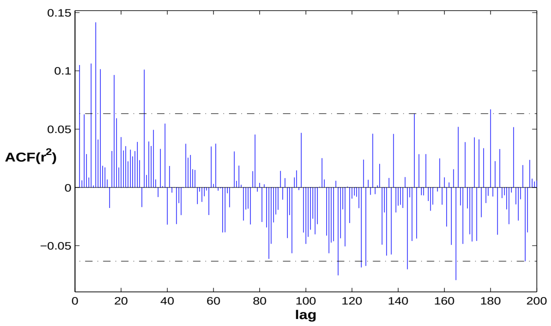
(c) The density of the return



(d) The ACs of the return



(e) The ACs of the absolute return



(f) The ACs of the squared return

FIGURE C.2. The time series of (a) the t price (red solid line) and the fundamental price (blue dot line) and (b) the return; (c) the density distribution of the returns; the ACs of (d) the returns; (e) the absolute returns, and (f) the squared returns, with the market noise only ($\sigma_\epsilon = 0$).

REFERENCES

Alfarano, S., Lux, T. and Wagner, F. (2005), 'Estimation of agent-based models: the case of an asymmetric herding model', *Computational Economics* **26**, 19–49.

Allen, H. and Taylor, M. (1990), 'Charts, noise and fundamentals in the London foreign exvhange market', *Economic Journal* **100**, 49–59. Conference.

Amilon, H. (2008), 'Estimation of an adaptive stock market model with heterogeneous agents', *Journal of Empirical Finance* **15**, 342–362.

Anufriev, M., and Hommes, C. (2012), 'Evolutionary selection of individual expectations and aggregate outcomes', *American Economic Journal – Micro* **4**, 35–64.

Baak, S. (1999), 'Test for bounded rationality with a linear dynamics model distorted by heterogeneous expectations', *Journal of Economic Dynamics and Control* **23**, 1517–1543.

Baillie, R. T., Bollerslev, T. and Mikkelsen, H. (1996), 'Fractionally integrated generalized autoregressive conditional heteroskedasticity', *Journal of Econometrics* **74**, 3–30.

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J. (2004), *Statistics of Extremes: Theory and Applications*, John Wiley & Sons Inc.

Bollerslev, T. (1986), 'Generalized autoregressive conditional heteroskedasticity', *Journal of Econometrics* **31**, 307–327.

Boswijk, H., Hommes, C. and Manzan, S. (2007), 'Behavioral heterogeneity in stock prices', *Journal of Economic Dynamics and Control* **31**, 1938–1970.

Brock, W. and Hommes, C. (1997), 'A rational route to randomness', *Econometrica* **65**, 1059–1095.

Brock, W. and Hommes, C. (1998), 'Heterogeneous beliefs and routes to chaos in a simple asset pricing model', *Journal of Economic Dynamics and Control* **22**, 1235–1274.

Cameron, A. and Trivedi, P. (2005), *Microeconometrics: Methods and Applications*, Cambridge University Press.

Canova, J. (1994), 'Statistical inference in calibrated models', *Journal of Applied Econometrics* **9**, 123–144.

Chavas, J. (2000), 'On the information and market dynamics: The case of the U.S. beef market', *Journal of Economic Dynamics and Control* **24**, 833–853.

Chen, S.-H., Chang, C.-L. and Du, Y.-R. (2012), 'Agent-based economic models and econometrics', *Knowledge Engineering Review* **27**, 187–219.

Chen, S.-H. and Yeh, C.-H. (2002), 'On the emergent properties of artificial stock markets: the efficient market hypothesis and the rational expectations hypothesis', *Journal of Economic Behavior and Organization* **49**, 217–239.

Cheung, Y.-W., Chinn, M. and Marxh, I. (2004), 'How do UK-based foreign exchange dealers think their market operates?', *International Journal of Finance and Economics* **9**, 289–306.

Chiarella, C. (1992), 'The dynamics of speculative behaviour', *Annals of Operations Research* **37**, 101–123.

Chiarella, C., Dieci, R. and Gardini, L. (2002), 'Speculative behaviour and complex asset price dynamics: a global analysis', *Journal of Economic Behavior and Organization* **49**, 173–197.

Chiarella, C., Dieci, R. and He, X. (2009), Heterogeneity, market mechanisms and asset price dynamics, *in* T. Hens and K. Schenk-Hoppe, eds, 'Handbook of Financial Markets: Dynamics and Evolution', North-Holland, Elsevier, pp. 277–344.

Chiarella, C. and He, X. (2002), 'An adaptive model on asset pricing and wealth dynamics with heterogeneous trading strategies', *Quantitative Finance Research Center, University of Techonology, Sydney* . Working Paper No. 84.

Chiarella, C. and He, X. (2003), 'Heterogeneous beliefs, risk and learning in a simple asset pricing model with a market maker', *Macroeconomic Dynamics* **7**, 503–536.

Chiarella, C., He, X. and Hommes, C. (2006), 'A dynamic analysis of technical trading rules in financial markets', *Journal of Economic Dynamics and Control* **30**, 1729–1753.

Chiarella, C., He, X., Huang, W. and Zheng, H. (2012), 'Estimating behavioural heterogeneity under regime switching', *Journal of Economic Behavior and Organization* **83**, 446–460.

Chiarella, C., He, X. and Zwinkels, R. (2014), 'Heterogeneous expectations in asset pricing: empirical evidence from the S&P 500', *Journal of Economic Behavior and Organization* **105**, 1–16.

Chiarella, C., ter Ellen, S., He, X. and Wu, E. (2015), 'Fear or fundamentals? Heterogeneous beliefs in the European sovereign CDS market', *Journal of Empirical Finance*, accepted, doi:10.1016/j.jempfin.2014.11.003.

Chung, C.-F. (1999), Estimating the fractionally integrated garch model, discussion paper, National Taiwan University.

Day, R. and Huang, W. (1990), 'Bulls, bears and market sheep', *Journal of Economic Behavior and Organization* **14**, 299–329.

De Grauwe, P. and Grimaldi, M. (2006), 'Exchange rate puzzles: A tale of switching attractors', *European Economic Review* **50**, 1–33.

de Jong, E., Verschoor, W. and Zwinkels, R. (2010), ‘Heterogeneity of agents and exchange rate dynamics: Evidence from the EMS’, *Journal of International Money and Finance* **29**(8), 1652–1669.

Dekkers, A., Einmahl, J. and de Haan, L. (1989), ‘A moment estimator for the index of an extreme-value distribution’, *Annals of Statistics* **17**, 1833–1855.

DeLong, J., Shleifer, A., Summers, L. and Waldmann, R. (1990), ‘Noise trader risk in financial markets’, *Journal of Political Economy* **98**, 703–738.

Diebold, F., L., O. and J., B. (1998), ‘Dynamic equilibrium economies: A framework for comparing models and data’, *Review of Economic Studies* **65**, 433–452.

Dieci, R., Foroni, I., Gardini, L. and He, X. (2006), ‘Market mood, adaptive beliefs and asset price dynamics’, *Chaos, Solitons and Fractals* **29**, 520–534.

Ding, Z., Granger, C. and Engle, R. (1993), ‘A long memory property of stock market returns and a new model’, *Journal of Empirical Finance* **1**, 83–106.

Dridi, R., Guay, A. and Renault, E. (2007), ‘Indirect inference and calibration of dynamic stochastic general equilibrium models’, *Journal of Economics* **136**, 397–430.

Dridi, R. and Renault, E. (2000), Semi-parametric indirect inference, EM, 392., Suntory and Toyota International Centres for Economics and Related Disciplines, London School of Economics and Political Science.

Engle, R. (1982), ‘Autoregressive conditional heteroscedasticity with estimates of the variance of uk inflation’, *Econometrica* **50**, 987–1008.

Farmer, J. and Joshi, S. (2002), ‘The price dynamics of common trading strategies’, *Journal of Economic Behavior and Organization* **49**, 149–171.

Franke, R. (2009), ‘Applying the method of simulated moments to estimate a small agent-based asset pricing model’, *Journal of Empirical Finance* **16**, 804–815.

Franke, R. and Westerhoff, F. (2011), ‘Estimation of a structural stochastic volatility model of asset pricing’, *Computational Economics* **38**, 53–83.

Franke, R. and Westerhoff, F. (2012), ‘Structural stochastic volatility in asset pricing dynamics: Estimation and model contest’, *Journal of Economic Dynamics and Control* **36**, 1193–1211.

Frankel, J. and Froot, K. (1990), ‘Chartists, fundamentalists and trading in the foreign exchange market’, *American Economic Review* **80**, 181–185.

Frijns, B. and T. Lehnert, R. Z. (2010), ‘Behavioral heterogeneity in the option market’, *Journal of Economic Dynamics and Control* **34**, 2273–2287.

Froot, K. A. and Dabora, E. M. (1999), ‘How are stock prices affected by the location of trade?’, *Journal of Financial Economics* **53**(2), 189–216.

Gaunersdorfer, A., Hommes, C. and Wagener, F. (2008), ‘Bifurcation routes to volatility clustering under evolutionary learning’, *Journal of Economic Behavior and Organization* **67**, 27–47.

Geweke, J. (1998), ‘Comment on real and spurious long memory properties of stock-market data (by lobato, i., and n. savin)’, *Journal of Business and Economic Statistics* **16**, 269–271.

Geweke, J. (2006), Computational experiments and reality, working paper, University of Iowa.

Geweke, J. and Poter-Hudak, S. (1983), ‘The estimation and application of long memory time series models’, *Journal of Time Series Analysis* **4**, 221–238.

Gilli, M. and Winker, P. (2003), ‘A global optimization heuristic for estimating agent-based model’, *Computational Statistics and Data Analysis* **42**, 299–312.

Goldbaum, D. and Mizrach, B. (2008), ‘Estimating the intensity of choice in a dynamic mutual fund allocation decision’, *Journal of Economic Dynamics and Control* **32**(12), 3866–3876.

Gourieroux, C., Monfort, A. and Renault, E. (1993), ‘Indirect inference’, *Journal of Applied Econometrics* **8**, 85–118.

Gray, G. and Kolda, T. (2006), ‘Algorithm 856: APPSPACK 4.0: Asynchronous parallel pattern search for derivative-free optimization’, *ACM Transactions on Mathematical Software* **32**(3), 485–507.

Griffin, J. and Kolda, T. (2006), Asynchronous parallel generating set search for linearly-constrained optimization, Technical report, Sandia National Laboratories, Albuquerque, NM and Livermore, CA.

Gromb, D. and Vayanos, D. (2010), ‘Limits of arbitrage’, *Annual Review of Financial Economics* **2**, 251–275.

Hansen, L. and Heckman, J. (1996), ‘The empirical foundations of calibration’, *Journal of Economic Prospectives* **10**, 87–104.

Hansen, L. P. and Jagannathan, R. (1997), ‘Assessing specification errors in stochastic discount factor models’, *Journal of Finance* **52**(2), 557–90.

He, X. (2003), Asset pricing, volatility and market behaviour-a market fraction approach, Technical Report 95, Quantitative Finance Research Center, University of Techonology, Sydney.

He, X. and Li, Y. (2007), ‘Power law behaviour, heterogeneity, and trend chasing’, *Journal of Economic Dynamics and Control* **31**, 3396–3426.

He, X. and Li, Y. (2008), ‘Heterogeneity, convergence and autocorrelations’, *Quantitative Finance* **8**, 58–79.

He, X.-Z. and Li, Y. (2015), 'Testing of a market fraction model and power-law behaviour in the DAX 30', *Journal of Empirical Finance* **31**, 1–17.

Heckman, J. (2001), 'Micro data, heterogeneity, and evaluation of public policy: Nobel lecture', *Journal of Political Economy* **109**(4), 673–748.

Hnatkovska, V., Marmer, V. and Tang, Y. (2012), 'Comparison of misspecified calibrated models: The minimum distance approach', *Journal of Econometrics* **169**(1), 131–138.

Hommes, C. (2001), 'Financial markets as nonlinear adaptive evolutionary systems', *Quantitative Finance* **1**, 149–167.

Hommes, C. H. (2006), Heterogeneous agent models in economics and finance, *in* L. Tesfatsion and K. L. Judd, eds, 'Handbook of Computational Economics', Vol. 2 of *Handbook of Computational Economics*, Elsevier, chapter 23, pp. 1109–1186.

Hommes, C., Sonnemans, J., Tuinstra, J. and Velden, H. v. d. (2005), 'Coordination of expectations in asset pricing experiments', *Review of Financial Studies* **18**, 955–980.

Kan, R. and Robotti, C. (2008), 'Specification tests of asset pricing models using excess returns', *Journal of Empirical Finance* **15**(5), 816–838.

Kan, R. and Robotti, C. (2009), 'Model comparison using the Hansen-Jagannathan distance', *Review of Financial Studies* **22**(9), 3449–3490.

Kolda, T. G. (2005), 'Revisiting asynchronous parallel pattern search for nonlinear optimization', *SIAM Journal on Optimization* **16**(2), 563–586.

Kydland, F. and Prescott, E. (1982), 'Time to build and aggregate fluctuations', *Econometrica* **50**, 1345–1370.

Kydland, F. and Prescott, E. (1991), 'The econometrics of the general equilibrium approach to business cycles', *Scandinavian Journal of Economics* **93**, 161–178.

Kydland, F. and Prescott, E. (1996), 'The computational experiment: an econometric tool', *Journal of Economic Prospectives* **10**, 69–86.

Lamont, O. A. and Thaler, R. H. (2003), 'Can the market add and subtract? Mispricing in tech stock carve-outs', *Journal of Political Economy* **111**(2), 227–268.

LeBaron, B. (2006), Agent-based computational finance, *in* L. Tesfatsion and K. L. Judd, eds, 'Handbook of Computational Economics', Vol. 2 of *Handbook of Computational Economics*, Elsevier, chapter 24, pp. 1187–1233.

Li, Y., Donkers, B. and Melenberg, B. (2006), The non- and semiparametric analysis of ms models: Some applications, CentER Discussion Paper 2006-95, Tilburg University.

Li, Y., Donkers, B. and Melenberg, B. (2010), 'Econometric analysis of microscopic simulation models', *Quantitative Finance* **10**, 1187–1201.

Lux, T. (1995), 'Herd behaviour, bubbles and crashes', *Economic Journal* **105**, 881–896.

Lux, T. (1998), 'The socio-economic dynamics of speculative markets: Interacting agents, chaos, and the fat tails of return distributions', *Journal of Economic Behavior and Organization* **33**, 143–165.

Lux, T. (2009*a*), 'Rational forecasts or social opinion dynamics? Identification of interaction effects in a business climate survey', *Journal of Economic Behavior and Organization* **72**, 638–655.

Lux, T. (2009*b*), Stochastic behavioural asset pricing and stylized facts, *in* T. Hens and K. Schenk-Hoppe, eds, 'Handbook of Financial Markets: Dynamics and Evolution', North-Holland, Elsevier, pp. 161–215.

Lux, T. (2012), 'Estimation of an agent-based model of investor sentiment formation in financial markets', *Journal of Economic Behavior and Organization* **36**, 1284–1302.

Lux, T. and Marchesi, M. (1999), 'Scaling and criticality in a stochastic multi-agent model of a financial markets', *Nature* **397**(11), 498–500.

Mandelbrot, B. (1963), 'The variation of certain speculative prices', *Journal of Business* **36**, 394–419.

Manzan, S. and Westerhoff, F. (2005), 'Representativeness of news and exchange rate dynamics', *Journal of Economic Dynamics and Control* **29**, 677–689.

Mehra, R. and Prescott, E. (1985), 'The equity premium: a puzzle', *Journal of Monetary Economics* **15**, 145–161.

Menkhoff, L. (1998), 'The noise trading approach — questionnaire evidence from foreign exchange', *Journal of International Money and Finance* **17**, 547–564.

Menkhoff, L. (2010), 'The use of technical analysis by fund managers: International evidence', *Journal of Banking and Finance* **34**, 2573–2586.

Pagan, A. (1996), 'The econometrics of financial markets', *Journal of Empirical Finance* **3**, 15–102.

Robinson, P. M. (1995), 'Log-periodgram regression of time series with long range dependence', *Annals of Statistics* **23**, 1048–1072.

Robinson, P. M. and Henry, M. (1999), 'Long and short memory conditional heteroscedasticity in estimating the memory parameters of levels', *Econometric Theory* **15**, 229–266.

Sargent, T. (1993), *Bounded Rationality in Macroeconomics*, Clarendon Press, Oxford.

Schorfheide, F. (2000), 'Loss function-based evaluation of DSGE models', *Journal of Applied Econometrics* **15**, 645–670.

Shefin, H. (2005), *A Behavioral Approach to Asset Pricing*, Academic Press Inc., London.

Shleifer, A. and Vishny, R. (1997), 'The limits of arbitrage', *Journal of Finance* **52**, 35–55.

Taylor, M. and Allen, H. (1992), 'The use of technical analysis in the foreign exchange market', *Journal of International Money and Finance* **11**, 304–314.

ter Ellen, S., Verschoor, W. and Zwinkels, R. (2013), 'Dynamic expectation formation in the foreign exchange market', *Journal of International Money and Finance* **37**, 75–97.

ter Ellen, S. and Zwinkels, R. (2010), 'Oil price dynamics: a behavioral finance approach with heterogeneous agents', *Energy Economics* **32**, 1427–1434.

Velasco, C. (1999), 'Non-stationary log-periodgram regression', *Journal of Econometrics* **91**, 325–371.

Vigfusson, R. (1997), 'Switching between chartists and fundamentals: a Markov regime-switching approach', *International Journal of Finance and Economics* **2**, 291–305.

Westerhoff, F. and Reitz, S. (2003), 'Nonlinearities and cyclical behavior: The role of chartists and fundamentalists', *Studies in Nonlinear Dynamics and Econometrics* **7**(4), article no. 3.

Winker, P. and Gilli, M. (2003), Indirect estimation of the parameters of agent based models of financial markets, Technical report, University of Geneva. CEF2003 paper.