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# Learning and Information Dissemination in Limit Order Markets

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# LEARNING AND INFORMATION DISSEMINATION IN LIMIT ORDER MARKETS

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Abstract. What can traders learn and how does learning affect the market? When information is asymmetric, short-lived, and uninformed traders learn, we present an artificial limit order market model to examine the effect of learning, information value, and order aggressiveness on information dissemination efficiency, bid-ask spread, order submission, and order profit of traders. We find that learning helps the uninformed traders to acquire private information more effectively and hence improves market information dissemination. Also the informed traders in general consume liquidity while the uninformed traders mainly supply liquidity. More interestingly, due to the learning and short-lived information, the bid-ask spread and its volatility are positively related to the probability of informed trading. The results help us to understand the behavior of uninformed traders and provide substantial insight and intuition into the trading process.

*Key words*: Limit order book, continuous double auction, learning, information dissemination, order aggressiveness, bid-ask spread.

*JEL Classification*: G14, C63, D82

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## 1. Introduction

Many financial markets around the world are limit order markets and the understanding of the price formation mechanism is one of the major goals in market microstructure. The traditional microstructure theory under a market maker (Kyle (1985)) has been extended recently to dynamic limit order market with asymmetric information, different information structure, and asset characteristics (Glosten (1994), Seppi (1994), Goettler, Parlour and Rajan (2009), and Rosu (2010)). The dynamic models developed reveal how information can be reflected in the market price through the interaction of informed traders and uninformed traders. Due to rapid development of internet and information technology, private information becomes short-lived and can be acquired by uninformed traders quickly through learning. Consequently, informed traders are expected to trade more aggressively (by submitting more market orders instead of limit orders) in order to benefit from their short-lived private information and inevitably release more private information to the market. At the same time, the unformed traders try their best to extract information from the market and to optimize their trading strategies in order to reduce their trading loss and increase their trading profit. Therefore, the decision-making of not only informed traders but also uninformed traders plays a key role in information dissemination and market behavior. To maintain tractability, most of microstructure models focus on the optimal trading strategies of informed traders by assuming that uninformed traders do not act strategically. However, as pointed out by O'Hara (2001), "*it is the uninformed traders who provide the liquidity to the informed, and so understanding their behaviors can provide substantial insight and intuition into the trading process.*" Furthermore, she puts forward an open question on what traders can learn from other pieces of market data, such as prices.

This paper aims to address this open question by presenting an artificial limit order market model of continuous double auction in which market is populated by informed traders who optimally trade on their information, uninformed traders who trade based on genetic algorithm (GA) learning, and zero-intelligence (ZI) traders. By focusing on the decision-making of uninformed traders, we examine the impact of learning on information dissemination efficiency and traders' order submission behavior, which in turn affect the bid-ask spread and order profit. This paper contributes to the literature in four aspects. (i) Consistent with the learning literature, the learning of the uninformed traders helps them to acquire the private information more effectively, which in turn improves market efficiency of information dissemination. (ii) The informed traders submit more aggressive orders and consume liquidity while the uninformed traders supply liquidity by submitting more limit orders. (iii) Different from the case when information is long-lived, the bid-ask spread and its volatility are positively related to the probability of informed trading (PIN) under learning and short-lived information. (iv) The informed traders gain from the uninformed traders due to the information advantage and the GA traders perform better than the ZI traders due to the learning. Some of the above effects become even more significantly with low information value, short information-lag, and less aggressiveness of the uninformed traders. Overall, we show that learning of the uninformed trader, together with information value and order aggressiveness, plays very important roles for market efficiency, dynamics of orders, and traders' behavior.

Information dissemination plays a very important role in price formation. In the continuous trading model of Kyle (1985), the optimal order quantity and expected profit of a rational representative informed trader are determined by the variance of the order flow of uninformed traders. The informed trader can hide his private information when the variance is large, leading to lower information dissemination efficiency. In Kyle's model, market makers set quotes rationally based on the observation of order flow and all private information is incorporated into prices at the end of trading. The static models (Glosten (1994) and Seppi (1994)) and dynamic models (Goettler et al. (2009) and Rosu (2010)) developed recently focus on exploring the roles of the informed traders while this paper is focused on the role of the uninformed traders in information dissemination process.

The role of uninformed traders in information dissemination process and the rich intraday phenomena driven by uninformed traders have been discussed in Admati and Pfleiderer (1988). Unlike Kyle (1985) that assumes uninformed traders are noisy and act randomly, Admati and Pfleiderer (1988) consider that some uninformed traders are discretionary, meaning that they can choose the timing of their transaction strategically. The discretionary uninformed traders concentrate their liquidity trade to reduce market impact, which attracts the informed traders to trade at the same pattern to maximize their profit. This helps to explain intraday phenomena such as "U shape" in trading volume. This illustrates the importance of uninformed traders trading strategies. However, as emphasized by O'Hara (2001), *"Neither sequential trade models such as Glosten and Milgrom (1985) nor batch trading models such as Kyle (1985) allow traders to learn anything from the movement of prices that is not already in their information set. But in actual asset markets the price elasticity of prices appears to be important. Technical analysis of market data is widespread in markets, with elaborate trading strategies devised to respond to the pattern of prices."* Due to the analytical difficulties, learning of uninformed traders in limit order markets has not been fully explored. In the absence of heterogeneity of uninformed traders, Goettler et al. (2009) use numerical simulation method to solve the equilibrium. Once we relax the strict assumption of analytic models, the set of decision states for investors can explode, becoming non-tractable analytically.

To overcome the challenge, this paper develops a model of limit order market with heterogeneous beliefs to explore the roles of uninformed traders in information dissemination efficiency, in particular, when the uninformed traders learn actively from market information.

Order aggressiveness is an important part of trading strategies when traders submit orders. It is closely related to the lived-time of private information. When investors are less (more) aggressive, they tend to submit limit (market) orders. Models with asymmetric information mainly focus on trading strategies of informed traders. The static models of Glosten (1994) and Seppi (1994) assume that information is short-lived and informed traders become more aggressive by using market order only. However, empirical studies (Bloomfield, O'Hara and Saar (2005), Kaniel and Liu (2006) and Menkhoff, Osler and Schmeling (2010)) find that informed and uninformed traders use both market and limit orders according to the market dynamics. In the dynamic model with short-lived asymmetric information in limit order markets, Goettler et al. (2009) find that the informed traders prefer to submit limit orders in a low volatility market but market orders in a high volatility market. Hence the market acts as a "volatility multiplier" that would cause a microstructure bias in the standard asset pricing model. Different from Goettler et al. (2009), Rosu (2010) assumes that information is long-lived and introduces waiting cost for submitting limit order so that the informed traders are patient and prefer to submit limit orders, while the uninformed traders can be patient or impatient and submit both limit orders and market orders. Rosu (2010) finds that a higher percentage of informed traders causes lower bid-ask spread and improves information dissemination efficiency. He proposes that the ratio of intraday price volatility to the average bid-ask spread can be used to estimate the probability of informed trading (PIN). With the limit order market model proposed in this paper, we are able to examine how the learning and order aggressiveness of the uninformed traders can affect order submission behavior of the informed traders and how bid-ask spread, its volatility, and the PIN are related.

This paper is largely motivated by the above literature and the modeling approach closely related to the recent development of heterogeneous agent-based models. This approach views the financial market as a complex adaptive system. It uses a bottom-up modeling approach to incorporate the interaction of adaptively heterogeneous behavior of traders and to examine the complex market behavior in aggregation. By considering the financial market as an expectations feedback mechanism, Chiarella (1992), Lux (1995) and Brock and Hommes (1998) were amongst the first to have shown that the interaction of agents with heterogeneous expectations may lead to market instability. By incorporating bounded rationality and heterogeneity, heterogeneous agent models have successfully explained the complexity of market price behavior, market booms and crashes, and long deviations of the market price from the fundamental price. They show some potentials in generating the stylized facts (such as skewness, kurtosis, volatility clustering and fat tails of returns), and various power laws (such as the long memory in return volatility) observed in financial markets. We refer the reader to Hommes (2006), LeBaron (2006), Lux (2009) and Chiarella, Dieci and He (2009) for surveys of the recent developments in this literature. This approach has been applied to model limit order markets. In limit order markets, stock price emerges from many individual investment decisions linked by information systems and social networks. To understand the decision-making and learning of uninformed traders, Chan, LeBaron, Lo and Poggio (2001) build a model with three types heterogeneous agents and study information dissemination in a limit order market. By assuming that agents only submit price-improving limit orders, they find that uninformed traders who use nearest-neighbor learning can improve information dissemination efficiency. To model the trading volume, Chiarella, Iori and Perellò (2009) extend the model of Chiarella and Iori (2002) and consider three types of heterogeneous agents, fundamentalists, chartists and noise traders. By allowing agents to use CARA utility to determine the trading volume under non-short sale constraint, they find that the chartist strategy is mainly responsible for the fat tails and clustering in the artificial price data generated by the model. More recently, Gil-Bazo, Moreno and Tapia (2007) build a model of informed and uninformed traders to include artificial neural network learning traders, trend traders and noise traders. They show that, when the information is long-lived, the artificial neural network learning traders can learn most of the private information from market data. See also the order driven behavior model of LiCalzi and Pellizzari (2003), models on the interaction of heterogeneous behavioral and market structure in Anufriev and Panchenko (2009), and on market efficiency under evolutionary learning with limited or full information in Anufriev, Arifovich, Ledyard and Panchenko (2013). These heterogeneous agent models show a great potential to examine the impact of the decision-making of uninformed traders on limit order markets. As Gould, Porter, Williams, Fenn and Howison (2012) suggest that, heterogeneity offered by agent-based models might pave the way for new explanations of rich order book phenomena.

This paper presents a limit order market model with asymmetric information and heterogeneous beliefs and study how leaning ability of uninformed traders influence information dissemination efficiency. Different from the rational expectations framework which employs Bayesian learning or Bellman function style updating learning (Goettler et al. (2009)), we consider an adaptive learning using genetic algorithm (GA). Firstly introduced by Holland (1975), GA has been widely used in economics and finance as an adaptive way to solve the investor's learning behavior (Arifovic (1994, 1996), Routledge (1999, 2001), and Chen (2002)). GA has also been used in Neo et al. (2003, 2006) to model two-side learning mechanism in corporate financing. A recently study using GA in limit order market includes Kluger and McBride (2011) in which informed and uninformed traders use GA to decide when to entry the market during a trading day and the model is able to generate some intraday trading patterns. This paper focuses on how uninformed traders use GA to learn from short-lived information and how the informed and uninformed traders interact via a limit order book.

The rest of paper is organized as follows. The model is outlined in Section 2. Section 3 describes experiment design and introduces performance measures. Section 4 examines the information dissemination efficiency. Section 5 focuses on order submission behavior and the impact on bid-ask spread and order profit. Section 6 concludes.

## 2. The Model

We consider a limit order book model of a single financial asset trading in a continuous double auction market. Traders are either informed or uninformed about the fundamental value of the asset. The informed traders know the information of the fundamental values when they enter the market, but not for the uninformed traders. The information is short-lived, meaning that the uninformed traders know the fundamental values with a time lag. The uninformed traders estimate (to be specified later) the expected fundamental value by using the lagged fundamental values and public information. Based on the expected fundamental values, traders may choose to buy or sell one share with the price at which they place their order. Transactions take place based on the standard price and time priorities in limit order markets. We now turn to the details of the model.

2.1. **Information.** The information structure is similar to Goettler et al. (2009). A trading time period $t$, defined by $(t-1, t]$, corresponds to a short time interval in the real market, such as one minute. The fundamental value $v_t$ of the risky asset at time period $t$ follows a random walk process. Innovations in the fundamental value $v_t$ occur according to a Poisson process with parameter $\phi$ and initial fundamental value $v_o$. If an innovation occurs, the fundamental value either increases or decreases with equal probability by $\kappa$ tick sizes. Depending on the value of parameter $\phi$, there may be more than one innovations in one time period, in particular when $\phi > 1$. In such case, the fundamental value $v_t$ of the time period $t$ is the last fundamental value of the time period. All the informed traders who enter the market in time period $t$ know the (same) fundamental value $v_t$; however the uninformed trader knows the fundamental value $v_{t-\tau}$ with a constant time lag $\tau > 0$ measured in units of time period. In general, the time lag $\tau$ can be different for different uninformed traders. In this paper, however, we keep the time lag the same for all uninformed traders but vary $\tau$ in different scenarios from 60 up to 1200 time periods (namely 1 hour up to 20 hours) to examine the effect of the information lag $\tau$.

2.2. **Traders.** There are $N$ risk neutral traders and each trader arrives at the market according to a Poisson process with parameter $\lambda$. To examine the effect of learning, traders are allowed to reenter the market on average every $\lambda$ time period, a more realistic feature in limit order book markets.[1] This provides the uninformed traders opportunities to learn from their trading. This is different from Goettler et al. (2009) where traders are allowed to reenter the market but only trade once, Also, to understand the impact of market structure and learning on the limit order book, the types of the traders are fixed, instead of choosing to be informed by paying for information in Goettler et al. (2009). A trader can submit a market order or a limit order (based on the rules specified later). Due to the reentry[2] of the traders to the market, the unexecuted limit orders are canceled after $\tau$ periods to reduce the pick-off risk, which is more realistic when $\tau$ is more than 60 time periods. Transaction may occur several times or none in one period. This is also different from Goettler et al. (2009) who model the cancelation endogenously to facilitate the trading of one share only for each trader. The limit order book at any point of time is a vector of the outstanding orders. Market price $p_t$ in time period $t$ is defined as the average of all the transaction prices during the time period. In case[3] if there is no transaction in the current time period $t$, we take the price of the last period as market price, $p_t = p_{t-1}$. The initial market price is equal to the initial fundamental value $p_o = v_o$. There is no price limit in the market. The cost of each transaction is fixed denoted by $\psi$. All traders observe the history of the transaction prices and order books.

We assume that there are $N_I$ and $N_U$ informed and uninformed traders, respectively, with $N_I + N_U = N$. To examine the impact of learning of the uninformed traders, we divide the uninformed traders into two groups with $N_{GA}$ genetic algorithm (henceforth GA) traders and $N_{ZI}$ zero-intelligence (henceforth ZI) traders with $N_{GA} + N_{ZI} = N_U$. Therefore the information value reflects the market structure of traders characterized by the proportions of different type of traders and a low (high) fraction of the informed traders corresponds to a market with high (low) information value. Intuitively, different information value has different impact on

[1] In general, the arriving rate for informed and uninformed traders can be different, which would affect the learning and limit order book differently.

[2] Effectively, this means that the order submission of each trader follows the Poisson process.

[3] This could happen when either $\lambda$ or $N$ is small.

the dynamics of the limit order book and market price. For comparison, we introduce a benchmark market structure based on some empirical evidence of the market information and noise ratios. Since the arrival rate of each type trader is assumed to be the same, the market *information ratio* measures the proportion of informed traders, while the market *noise ratio* represents the proportion of ZI traders.

When trader $i$ enters the market at time $t' \in (t-1, t]$ in time period $t$, he compares the latest market price $p_{t-1}$ with the expected fundamental value $p^i_{t'} := E^i_{t'}(v_t)$ and submits an order accordingly. The informed trader $i$ knows the fundamental value when he enters the market, so his expected fundamental value in time period $t$ is given by $p^i_{t'} = v_t$. The uninformed trader knows the lagged fundamental value $v_{t-\tau}$ when he enters the market and estimates the expected fundamental value based on two market variables:[4] the average market price $\bar{p}_{t,\tau}$ over the last $\tau$ periods and the mid-price $p^m_{t'}$ of the current bid $b_{t'}$ and ask $a_{t'}$ prices, that is,

$$\bar{p}_{t,\tau} = \frac{1}{\tau}[p_{t-1} + p_{t-2} + \cdots + p_{t-\tau}], \qquad p^m_{t'} = \frac{1}{2}(a_{t'} + b_{t'}).$$

More specifically, we follow Chiarella, Iori and Perellò (2009) and assume that the expected fundamental value in the time period $t$ for the uninformed trader $i$ is given by

$$p^i_{t'} = E^i_{t'}(v_t) = \frac{1}{\alpha^i_{t'} + \beta^i_{t'} + \gamma^i_{t'}}(a^i_{t'} v_{t-\tau} + \beta^i_{t'}\bar{p}_{t,\tau} + \gamma^i_{t'} p^m_{t'}), \tag{1}$$

where $\alpha^i_{t'}, \beta^i_{t'}$ and $\gamma^i_{t'}$ are forecasting parameters of the uninformed trader. For the GA traders, they optimize these parameters by using genetic algorithm (see details in section 2.5). For the ZI traders, they choose these parameters randomly based on some distributions. Different from the existing literature of zero-intelligence or noise trader models, the ZI traders in the model use the market information.

2.3. **Order Aggressiveness.** The order aggressiveness is closely influenced by the structure of asymmetric information and how uninformed traders explore the private information. When the information is long-lived, the uninformed traders do not have the history and currently private information about the fundamental values and they can only learn from market information. When information is short-lived, the uninformed traders know the history private information after some time lag $\tau$. Empirically, Menkhoff et al. (2010) use the trading activity to distinguish informed traders from uninformed traders. They find that informed traders use aggressive limit orders (limit sell order below the best ask or limit buy order above the best bid) to replace market orders, and uninformed traders use aggressive limit orders to replace less aggressive limit orders (limit sell order above the best ask or limit

[4]The uninformed traders may use other market information including the type of last order, trading volume, order book depth and shape, etc. To simplify the analysis of the impact of learning on information dissemination efficiency, we only focus on these two market variables in this paper.

buy order below the best bid). In an equilibrium model, Rosu (2010) shows that, when the information is long-lived, the informed traders submit market orders when the market price deviate from the fundamental value significantly and limit orders otherwise. When the information is short-lived, Goettler et al. (2009) obtain a similar result that the informed traders prefer to submit market orders in a high volatility market and limit orders in a low volatility market.

In this paper we follow Goettler et al. (2009) and assume that the information is short-lived. This implies that the informed traders are impatient and they prefer to submit more aggressive order in order to benefit from the short-lived information. Instead, the uninformed traders are patient in order to reduce their trading loss to the informed traders and therefore they require a liquidity compensation $\omega$ for offering limit orders. The liquidity compensation $\omega$ could include opportunity cost and monitoring and learning costs. A large $\omega$ means that the uninformed traders are more patient and submit less aggressive orders, and they may adjust their order aggressiveness when market condition changes. Therefore the liquidity compensation $\omega$ is a measure of the order aggressiveness of the uninformed traders. Clearly, when information is short-lived, both the time lag $\tau$ and the liquidity compensation $\omega$ affect the trading behavior of the uninformed traders. In addition, the transaction cost $\psi$ can also affect the order aggressiveness of traders. Hence the order aggressiveness of the uninformed traders can be measured by $\mu = \omega + \psi$. In particular, $\mu = \psi$ measures the order aggressiveness of the informed traders. The high the parameter $\mu$ is, the less aggressive the traders are.

2.4. **Order Submission Rules.** Traders trade only when the expected order profit from trading is high enough to offset the transaction cost. In a dynamic equilibrium model of an order driven market with asymmetric information, Rosu (2010) shows that informed traders submit both market orders and limit orders, depending on whether their informative advantage is about a cutoff. In an agent-based model, Gil-Bazo et al. (2007) has introduced similar order submission rules for all risk neutral and myopic traders who submit market orders when the price level diverges far from their forecasting fundament value and limit orders when the deviation is small. Depending on traders' forecasting of market price and the order book states, traders submit either limit or market orders. We introduce similar order submission rules as in Gil-Bazo et al. (2007) and Rosu (2010). When trader $i$ arrives in the market at time $t'$ in time period $t$, he compares his expected fundamental value $p^i_{t'}$ with the current best bid $b_{t'}$ and best ask $a_{t'}$, together with his order aggressiveness $\mu$. Depending on the current order book, there are four scenarios summarized in Table 1. For example, in the first case, there is at least one ask and one bid in the current limit order book. In this case, a trader submits a market order to buy when his expected fundamental value $p^i_{t'}$ is above the sum of the best ask $a_{t'}$ and the order aggressiveness $\mu$, that is $p^i_{t'} > a_{t'} + \mu$. When his expected fundamental value $p^i_{t'}$ is below the best bid $b_{t'}$, less the order aggressiveness $\mu$, that is $p^i_{t'} < b_{t'} - \mu$, he submits a market order to sell. When $b_{t'} - \mu \leq p^i_{t'} \leq a_{t'} + \mu$, he submits a limit buy or sell order, depending on whether the expected fundamental value $p^i_{t'}$ is above or below the current mid-price $p^m_{t'} = (a_{t'} + b_{t'})/2$. The order submission rules for the other three cases are defined similarly.

| Scenario | Order |
|---|---|
| *Case 1: There is at least one ask price and one bid price in the limited order book* | |
| $p_{t'}^{i} > a_{t'} + \mu$ | Market order to buy |
| $a_{t'} + \mu \geq p_{t'}^{i} \geq b_{t'} - \mu \& \lvert a_{t'} - p_{t'}^{i} \rvert \leq \lvert p_{t'}^{i} - b_{t'} \rvert$ | Limit order to buy at $p_l = p_{t'}^{i} - \mu$ |
| $a_{t'} + \mu \geq p_{t'}^{i} \geq b_{t'} - \mu \& \lvert a_{t'} - p_{t'}^{i} \rvert > \lvert p_{t'}^{i} - b_{t'} \rvert$ | Limit order to sell at $p_l = p_{t'}^{i} + \mu$ |
| $p_{t'}^{i} < b_{t'} - \mu$ | Market order to sell |
| *Case 2: There is no bid price* | |
| $p_{t'}^{i} > a_{t'} + \mu$ | Market order to buy |
| $p_{t'}^{i} \leq a_{t'} + \mu$ | Limit order to buy at $p_l = p_{t'}^{i} - \mu$ |
| *Case 3: There is no ask prices* | |
| $p_{t'}^{i} < b_{t'} - \mu$ | Market order to sell |
| $p_{t'}^{i} \geq b_{t'} - \mu$ | Limit order to sell at $p_l = p_{t'}^{i} + \mu$ |
| *Case 4: There is no ask or bid price* | |
| With probability 50% | Limit order to buy at $p_l = p_{t'}^{i} - \mu$ |
| With probability 50% | Limit order to sell at $p_l = p_{t'}^{i} + \mu$ |

TABLE 1. Order submission rules, where $\mu = \omega + \psi$ for the uninformed and $\mu = \psi$ for the informed traders.

2.5. **Artificial Stock Market and GA Learning.** The artificial limit order market considered in this paper is based on FinancialMarketModel[5] and implemented

[5]FinancialMarketModel is a framework of several artificial stock markets. It is developed by CSS 739 Team under direction of Robert Axtell. We refer the details to http://www.assembla.com/wiki/show/MarketModel.

using the MASON simulation toolkit. The FinancialMarketModel uses a continuous double auction trading mechanism and the design of the order book is based on Farmer, Patelli, Zovko and Arrow (2005). We name the artificial model as CDA-ASM since it employs *continuous double auction* (CDA) trading mechanism in an *artificial stock market* (ASM). The key task of developing CDA-ASM is the implementation of genetic algorithm (GA).

GA uses chromosomes to represent forecasting rules or trading strategies, and then evolves chromosomes by selection, crossover, and mutation processes. It is introduced firstly by Holland (1975) as an optimal adaptive learning algorithm based on natural selection. From the evolution perspective, GA has two adaptive types. One type uses a classifier system to describe world conditions, which, together with a set of special forecasting parameters, generate forecasting rules. Each agent has a set of forecasting rules. When an agent is active, he chooses the best forecasting rule corresponding to the current world conditions for the action. A simple description is that when the world condition $x$ happens, the agent chooses the best forecasting rule $y_x^*$. The forecasting parameters of the forecasting rule evolve according to the performance of the forecasting rule. Agents indirectly interact via the market environment. A typical application of this type GA is the Santa Fe Institute Artificial Stock Market (SFI-ASM) in Arthur, Holland and Palmer (1997). Another type does not use classifier system and its chromosomes evolve according to other agents' strategies. Strategies of different type agents have some specifical correlations. For example, if agent A chooses a strategy $X$, then agent B should choose a relative strategies $Y_X^*$. This paper focuses on how the uninformed traders learn from market information. It allows informed and uninformed traders interact via a limit order book. Hence the first type GA with classifier system well captures the learning behavior of the uninformed trader. Based on equation (1), the GA traders learn from historical price and mid-price, and adapt to market condition. We use two key ratios, $p_{t'}^m/\bar{p}_{t,\tau}$ and $\bar{p}_{t,\tau}/v_{t-\tau}$, to classify the market conditions and compare these two factors across different time range similar to technical analysis rules. The classifier system generates a number of classify rules for a number of market conditions. The forecasting rules are constructed by market conditions and the corresponding forecasting parameters $\alpha_{t'}^i, \beta_{t'}^i$ and $\gamma_{t'}^i$ specified in equation (1). The forecasting rules are then optimized by genetic algorithm through selection, crossover and mutation. The details of the GA learning is given in Wei, Zhang, He and Zhang (2013) who show that the model developed in this paper is able to generate a number of important stylized facts, including leptokurtosis, fat tails, volatility clustering, and long memory, together with some limit order book phenomena discussed in the Goettler et al. (2009). Hence, the CDA-ASM model of this paper provides a reasonable framework for studying the learning and information dissemination in limit order markets.

## 3. Experiment Design and Performance Measures

The limit order market model introduced in the previous section is designed to analyze the impact of learning, information-lived time, information value, and order aggressiveness on the efficiency of information dissemination. For comparison, we first consider a benchmark model (BM). To quantify the impact, we also introduce some performance measures in this section.

3.1. **A Benchmark Model.** For the benchmark model (BM), as in Goettler et al. (2009), we normalize the mean arriving time of trader $\lambda = 1$. If one time period corresponds to one minute, this implies that, on average, each trader enters the market once in every minute. We set the tick size $\delta = 0.01$ and the transaction cost $\psi = 0.04$, which is four ticks. Let the initial fundamental value $v_o = 20$. Then the variation of the fundamental value is 0.4 per 10 minutes on average.[6] In the BM, we assume that $\phi = 4$ and $\kappa = 1$. This implies that, on average, the innovation of the fundamental value occurs four times per minute and each innovation changes the fundamental value by one tick size.

To speed up the simulations, we assume that there are $N = 100$ traders. Among which, there are $N_I = 12$ informed traders, $N_{GA} = 30$ GA traders, and $N_{ZI} = 58$ ZI traders in the BM.[7] The uninformed traders observe the fundamental value with a lag of $\tau = 120$ periods, corresponding to two hours or half trading day ((based on four trading hours per trading day in the Chinese stock markets). The time lag varies from 60 to 1,200 in different scenarios, corresponding to one to 20 hours, which are much longer than the time lags of 16 to 128 time periods, corresponding to about 1/4 to two hours, used in Goettler et al. (2009). This difference reflects the different learning mechanisms. In Goettler et al. (2009), each trader has only one share to trade and leaves the market after the trading. Therefore the time lag is relatively short so that the uninformed traders are able to learn and trade. In our model, traders have opportunity to reentry the market and learn from lagged fundamental values over a relative long time lags. Finally, we set the minimum and maximum values of parameters $\alpha_t^i, \beta_t^i$ and $\gamma_t^i$ in equation (1) to be 0.01 and 0.99,

[6]In Goettler et al. (2009), the expected change in the fundamental value is about 2.5 ticks per 10 minutes in low volatility case. With the tick size of 1/8, this is about 0.31. In our model, the expected change is 0.40.

[7]As suggested by Rosu (2010), the fraction of informed traders can be used to estimate the PIN, so we use the PIN as a proxy of informed traders' proportion. We also use the noise ratio to represent the proportion of the ZI traders. On the empirical studies of Shanghai stock market in China, Wang, Zhang and Fang (2009) find that the PIN is between 11.21% and 18.62% and Xu and Liu (2009) find the probability of noise trading is about 58.14%. Accordingly, we set the proportions to be 12%, 58% and 30% for the informed, ZI, and GA traders respectively, in the benchmark model.

respectively. The order aggressiveness for the BM model is assumed to be $\omega = 0$ for all the traders. The main parameter setting for the BM model is collected in the BM scenario/experiement in Table 2.

With the set of parameters, we run 30 simulations for statistical significance. Since the GA traders need sufficient learning time to obtain optimal forecasting rules, each simulation runs 60,000 periods, but the analysis is based on the results from 48,001 to 60,000 periods, in total of $T = 12,000$ periods, which is about 200 hours or about 50 trading days.

| Scenario | Experiment | $N_I$ | $N_{GA}$ | $N_{ZI}$ | $\tau$ | $\omega_{GA}$ | $\omega_{ZI}$ |
|---|---|---|---|---|---|---|---|
| BM | BM | 12 | 30 | 58 | 120 | 0 | 0 |
| I | A1 | 12 | 0 | 88 | 120 | 0 | 0 |
| | A2 | 12 | 88 | 0 | 120 | 0 | 0 |
| II | B1 | 12 | 30 | 58 | 60 | 0 | 0 |
| | B2 | 12 | 30 | 58 | 240 | 0 | 0 |
| | B3 | 12 | 30 | 58 | 1200 | 0 | 0 |
| III | D1 | 35 | 30 | 35 | 120 | 0 | 0 |
| | E1 | 1 | 30 | 69 | 120 | 0 | 0 |
| | F1 | 69 | 30 | 1 | 120 | 0 | 0 |
| IV | C2 | 12 | 30 | 58 | 120 | 0.04 | 0.04 |
| | C3 | 12 | 30 | 58 | 120 | 0.04 | 0 |
| | C4 | 12 | 30 | 58 | 120 | 0 | 0.04 |
| | D2 | 35 | 30 | 35 | 120 | 0.04 | 0.04 |
| | D3 | 35 | 30 | 35 | 120 | 0.04 | 0 |
| | D4 | 35 | 30 | 35 | 120 | 0 | 0.04 |
| | E2 | 1 | 30 | 69 | 120 | 0.04 | 0.04 |
| | E3 | 1 | 30 | 69 | 120 | 0.04 | 0 |
| | E4 | 1 | 30 | 69 | 120 | 0 | 0.04 |
| | F2 | 69 | 30 | 1 | 120 | 0.04 | 0.04 |
| | F3 | 69 | 30 | 1 | 120 | 0.04 | 0 |
| | F4 | 69 | 30 | 1 | 120 | 0 | 0.04 |

TABLE 2. The parameters of all the scenarios.

3.2. **The Learning.** To examine whether the ability of the GA traders strengthens their information acquisition and improves market information dissemination efficiency, we consider two special cases in which the uninformed traders are either ZI traders or GA traders only by keeping the number of the informed traders fixed as in the BM model. They correspond to Scenario I, experiments A1 and A2, respectively, of Table 2. When forming the expected fundamental values, both GA and ZI traders use a weighted average of the current mid-price, the price moving average, and the lagged fundamental values described in equation (1). The difference is that the weighting coefficients are random for the ZI traders, but updated through learning for the GA traders. The comparison among the two cases, together with the benchmark case, can provide some insights into the learning ability of GA traders and their impact on the market.

3.3. **Information-lived Time.** When the information is short-lived, the uninformed traders know the fundamental value with a time lag of $\tau$. In real market, the time lag $\tau$ may change with traders' learning ability, media coverage, technologies, and market conditions. With time lag $\tau = 60, 240$ and 1,200 time periods in Scenario II of Table 2, together with $\tau = 120$ in the BM model, we can examine the impact of the information-lived time lag on information dissemination efficiency through experiments BM, B1, B2, and B3.

3.4. **Information Value.** Information value depends on the market fraction of the informed traders. When the fraction is low (high), the information value is high (low) and the market become less (more) informative. Plausibly, market fractions of different types of traders can have significant impact on the information dissemination efficiency, in particular, when the market is dominated by one type of traders. To examine such impact, we consider three different market structures $D1, E1$ and $F1$ for different combinations of the informed and ZI traders with the fixed number of the GA traders in Scenario III of Table 2, together with the BM case. The different market structure represents different market information ratio, which measures the proportion of the informed traders, and the market noise ratio, which measures the proportion of the ZI traders. A high information ratio indicates that there are more informed traders, market is more informative, and the information is less valuable. A high noise ratio indicates that there are more noise traders, market is less informative, and the information is more valuable. We consider two extreme cases, E1 and F1 in which either the information ratio or noise ratio is very low, and two normal cases, BM and D1, to examine the effect of the market structure.

3.5. **Order Aggressiveness.** The order aggressiveness is measured by $\mu$, which is $\psi$ for the informed traders and $\psi+\omega$ for the uninformed traders. A high $\mu$ implies less aggressiveness. To examine the impact of the order aggressiveness of the uninformed traders on information dissemination efficiency, we choose $\psi = 0.04$ and select $\omega$ to be either 0 or 4 tick sizes, leading to four combinations of the order aggressiveness $(\omega_{GA}, \omega_{ZI}) = (0,0), (0,0.04), (0.04,0)$ and $(0.04, 0.04)$ for the uninformed traders. Combined with four market structures, they are grouped into C, D, E and F groups in Scenario IV of Table 2.

3.6. **Performance Measures.** The information dissemination efficiency is measured by the convergence of the market price to the fundamental value. Following Theissen (2000), we introduce two performance measures.

**Information efficiency measure**, which measures the information content of the trading price. We use Mean Absolute Error (MAE) to measure the deviation or error of the market price $p_t$ or mid-price (midpoint of the quoted prices) $p_t^m$ from the fundamental value $v_t$,

$$MAE_p = \frac{1}{T}\sum_{t=1}^{T}|p_t - v_t|, \qquad MAE_{p^m} = \frac{1}{T}\sum_{t=1}^{T}|p_t^m - v_t|. \tag{2}$$

In the literature, the mid-price is often used to measure the market efficiency. However, Goettler, Parlour and Rajan (2005) point out that the market price is a better proxy for the fundamental value than the mid-price. We also use Mean Relative Error (MRE) to measure the relative error of the market price from the fundamental value,

$$MRE = \frac{1}{T}\sum_{t=1}^{T}\frac{|p_t - v_t|}{v_t}. \tag{3}$$

**Information acquisition measure**, which measures how a trader can explore the fundamental information efficiently based on trader's prediction error of the expected fundamental value from the fundamental value. In our model, this error is always zero for the informed traders. For the uninformed traders, the error of trader $i$ is defined by the Mean Absolute Deviation (MAD) over $T$ time periods,

$$MAD_i = \frac{1}{T}\sum_{t=1}^{T}|p_t^i - v_t|. \tag{4}$$

We also examine the profitability of different types of traders. The order profit is calculated by the order return $r_t$ based on the difference between the transaction price of the order $p_t$ and the fundamental value $v_t$. It is $r_t = v_t - p_t$ for an executed buy order and $r_t = p_t - v_t$ for an executed sell order. We then denote $r_I$, $r_{GA}$ and $r_{ZI}$ the average order profit of the informed, GA, and ZI traders, respectively; we also denote $R_I$, $R_{GA}$ and $R_{ZI}$ the total order profit of the informed, GA, and ZI traders, respectively.[8]

In the next section, we use the performance measures to examine the impact of learning, information-lived time, information value, and order aggressiveness on information dissemination efficiency. In section 5, we examine their impact on order submission, bid-ask spread, and order profit.

## 4. Information dissemination efficiency

This section examines the information dissemination efficiency by considering four scenarios, which focus on four different aspects discussed in Section 3. The parameter set up of the four scenarios is summarized in Table 2.

4.1. **Scenario I: The effect of learning.** In this scenario, the number of the informed traders $N_I = 12$ is fixed as in the BM, while the numbers of the GA and ZI traders are either $N_{GA} = 0, N_{ZI} = 88$ in case $A1$ or $N_{GA} = 88, N_{ZI} = 0$ in case $A2$. The results of the information efficiency and acquisition measures are presented in Table 3, comparing to the BM case of $N_{GA} = 30, N_{ZI} = 58$. A further ANOVA analysis on the MAE using the traded price for the three cases is presented in Fig. A.1 in the appendix.

| Experiment | $MAE_p$ | $MAE_{pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|
| BM | 0.1465 | 0.1577 | 0. 78% | 0.1593 | 0.1691 |
| A1 | 0.1560 | 0.1668 | 0. 86% | None | 0.1754 |
| A2 | 0.1320 | 0.1450 | 0. 65% | 0.1521 | None |

Table 3. The impact of learning on the information efficiency and acquisition.

The results in Table 3 show that the information dissemination efficiency improves when there are more GA traders in the market. Consistently across all three scenarios, all the MRE and the MAE in Table 3 using either the market price or the mid-price are the smallest for $A2$ and largest for $A1$. The ability of information acquisition is demonstrated by that the MAD is smaller for the GA traders (in $A2$) and larger for the ZI traders (in $A1$), comparing with the MAD in the BM case. Hence the ability of information acquisition improves for the GA traders when they dominate the ZI traders, but becomes worse for the ZI traders when they dominate the GA traders. In addition, one can see that $MAE_p < MAE_{p^m}$ consistently across

[8]Since each trader only trades one share and trading is a zero-sum game, the transaction cost is the same for for all the traders, so we do not consider the transaction cost when calculating the returns.

all the cases. This indicates that the market price is more informative than the mid-price, which is consistent with the result of Goettler et al. (2005)). In conclusion, the results show that

(i) *when the information is short-lived, the GA traders acquire the private information effectively through learning, which in turn improve the information dissemination in the market.*

4.2. **Scenario II: The effect of information lived-time.** Most likely, the shorter the information lag is, the more quickly the uninformed traders acquire the information, the more efficient the market prices become. This intuition is confirmed by the results in Table 4 where we consider three different information lived-time $\tau = 60, 240$ and 1,200 in scenario II, together with $\tau = 120$ in the BM case.

| Experiment | $\tau$ | $MAE_p$ | $MAE_{pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|
| B1 | 60 | 0.0959 | 0.1086 | 0. 48% | 0.1081 | 0.1172 |
| BM | 120 | 0.1465 | 0.1577 | 0. 78% | 0.1593 | 0.1691 |
| B2 | 240 | 0.2172 | 0.2269 | 1.23% | 0.2285 | 0.2409 |
| B3 | 1200 | 0.5480 | 0.5639 | 2.97% | 0.5406 | 0.5914 |

TABLE 4. The effect of information lived-time on the information efficiency and acquisition.

Based on Table 4, we can see that MAE, using either the market price or mid-price, increases as $\tau$ increases from 60 (case $B1$), to 120 (BM case), to 240 (case $B2$), and then to 1200 (case $B3$). Comparing the MAD among the 4 cases, we see that the prediction error for both the GA and ZI traders becomes smaller as $\tau$ decreases. Thus we have the following observation.

(ii) *The information acquisition of the uninformed traders becomes more effective and information dissemination efficiency improves when the information lived-time lag decreases.*

Interestingly, due to the learning, the GA traders acquire more information advantage than the ZI traders, in particular when $\tau = 1200$ (see Table 4, the difference between $MAD_{GA}$ and $MAD_{ZI}$ is the biggest in experiment B3). That is because when the information lag becomes longer, although both GA and ZI traders have the same information about the fundament value, but the GA traders learn from more market information on the history prices. This also implies that the technical analysis is useful for the uninformed traders.[9] Hence, when traders learn from more

[9]The classifier system of genetic algorithm employs some technical rules, we refer to Wei et al. (2013) for more details.

market information, such as trading volume, order book depth and shape, we expect the ability of the uninformed traders to acquire the information can be improved significantly. In addition, MAD can be used to measure the cost for purchasing information, which may depend on the learning ability of traders. A smart trader may learn the information at a low cost, which reduces the information cost. Therefore it would be interesting to examine the trade-off between information cost and learning cost.

4.3. **Scenario III: The effect of information value.** The information value in a market plays a very important role. It can be measured by the ratio of the numbers of informed and uninformed traders in the market. A low information value means more informed traders, leading to better information dissemination efficiency. Also, due to the learning of the GA traders, we expect the information to be disseminated more efficiently when there are more GA traders comparing to the ZI traders. In scenario III, we keep the number of GA traders fixed at $N_{GA} = 30$ and change $N_I$, the number of the informed traders, from 12 in the BM case to 35 in D1, 1 in E1, and 69 in F1. The results are reported in Table 5, which lead to the following observation.

(iii) *The information dissemination efficiency, measured by the decreasing in MAE, MRE and MAD, improves as the number of informed traders increases and the number of ZI trades decreases.*

| Experiment | Proportion | $MAE_{Pt}$ | $MAE_{Pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|
| E1 | 1:30:69 | 0.1719 | 0.1738 | 0.93% | 0.1659 | 0.1778 |
| BM | 12:30:69 | 0.1465 | 0.1577 | 0.78% | 0.1593 | 0.1691 |
| D1 | 35:30:35 | 0.1079 | 0.1364 | 0. 60% | 0.1413 | 0.1512 |
| F1 | 69:30:1 | 0.0636 | 0.0995 | 0. 32% | 0.1046 | 0.1271 |

TABLE 5. The effect of information value on the information efficiency and acquisition.

4.4. **Scenario IV: The effect of order aggressiveness.** The order aggressiveness $\omega$ reflects the liquidity compensation for the opportunity or learning cost of the uninformed traders. To examine the effect of order aggressiveness, we consider four cases with $(\omega_{GA}, \omega_{ZI}) = (0, 0), (0.04, 0.04), (0.04, 0)$ and $(0, 0.04)$. A non-zero $\omega$ means that the traders are less aggressive. Together with the BM case, we present the results in Table 6. A further analysis on Experiment Groups D, E and F and their comparison can be found in Figure 1 and Tables A.1, A.2, and A.3 in the appendix.

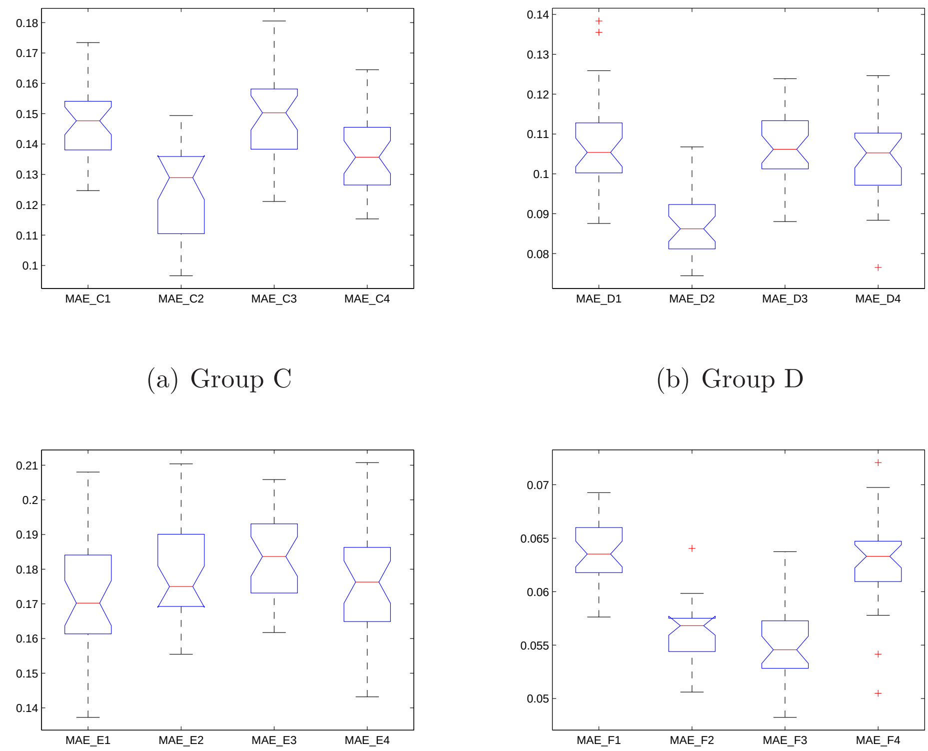
| Experiment | $\omega_{GA}$ | $\omega_{ZI}$ | $MAE_{Pt}$ | $MAE_{Pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|---|
| BM | 0 | 0 | 0.1465 | 0.1577 | 0. 78% | 0.1593 | 0.1691 |
| C2 | 0.04 | 0.04 | 0.1233 | 0.1493 | 0. 70% | 0.1534 | 0.1630 |
| C3 | 0.04 | 0 | 0.1506 | 0.1606 | 0. 79% | 0.1606 | 0.1709 |
| C4 | 0 | 0.04 | 0.1363 | 0.1492 | 0. 77% | 0.1555 | 0.1658 |

TABLE 6. The effect of order aggressiveness on the information efficiency and acquisition.

The results in Table 6 show that, when both the GA and ZI traders become less aggressive, all the MAE for case C2 are the smallest among the four cases. Consequently, the uninformed traders become less active in the market. The informed traders are thus more active, which improve the information efficiency, leading to small deviation of the market price from the fundamental price. For case C3 where only the GA traders are less aggressive, the model produces the highest MAE. Comparing to the BM case, the information efficiency declines. Intuitively, when the GA traders become less active, they contribute less to information dissemination. Consequently, the deviation of the market price from the fundamental price becomes large. Since the GA traders obtain more information than the ZI traders from their learning, one would expect the opposite when the ZI traders become less aggressive. This is indeed the case shown by the results for case C4 in Table 6. This observation also holds when there are more informed traders and less ZI traders (with the same number of the GA traders) shown in Table A.1 in the appendix. The only difference is that, when there are more informed traders, the effect of the order aggressiveness of the GA traders on the information dissemination efficiency become weaker (cases D1 and D3). The above results can be summarized as follows.

(iv) *Both information dissemination efficiency and acquisition are improved when the uninformed traders submit orders less aggressively, however the effect of order aggressiveness to the information dissemination efficiency is opposite for the GA and ZI traders. Also, market information become more efficient when there are more informed traders and the benefit from learning becomes less significantly.*

For the two extreme market structures, we report the results in Table A.2 when there is only one informed trader (Group E) and in Table A.3 when there is only one ZI trader (Group F) in the appendix. For the lowest informative market E, comparing the results among the four cases, we observe that, when all the uninformed trader reduce their order aggressiveness, the MAE increases (Case E2). When only




(a) Group C (b) Group D

(c) Group E (d) Group F

FIGURE 1. An ANOVA analysis of the effect of order aggressiveness on the MAE. Here C1 in Group C denotes BM.

the GA traders become less aggressive, the MAE increases significantly (Case E3). When only the ZI traders become less aggressive, the MAE does not change significantly. Intuitively, when there is only one informed trader, the information is disseminated mainly through the learning of the GA traders and their market order and aggressive limit orders. When the GA traders reduce their order aggressiveness, the information dissemination declines (Cases E2 and E3). For the highly informative market F, the market is dominated by the informed traders while the ZI traders have insignificant influence on the market information dissemination (Case F4). Relatively, in this extreme case, the GA trader can be considered as noise traders. When the GA traders reduce their order aggressiveness, the information dissemination improve slightly (Cases F2 and F3). Hence, the GA traders play opposite roles in the two extreme cases. This is further verified by the results in Fig. 1 in which the dissemination efficiency is low (with high MAE) for E2 and E3 in Fig. 1 (c) and high (with low MAE) for F2 and F3 in Fig. 1 (d) when the GA traders reduce their order aggressiveness.

In conclusion, in normal market where the information and noise ratios are not extreme, the information dissemination efficiency improves when all the uninformed traders become less aggressive. However, except market structure F with only one ZI trader, the order aggressiveness of the GA traders and ZI traders has opposite impact on the information dissemination efficiency.

## 5. Order Submission, Bid-ask Spread and Order Profit

In this section, we examine the effect of learning, information value, and order aggressiveness of traders on order submission, bid-ask spread, and order profit.

| Experiment | ILO | IMO | ILE | GALO | GAMO | GALE | ZILO | ZIMO | ZILE |
|---|---|---|---|---|---|---|---|---|---|
| BM | 38,251 | 105,701 | 2,576 | 259,344 | 12,422 | 38,573 | 527,131 | 39,468 | 116,442 |
| C2 | 50,903 | 93,159 | 1,262 | 285,788 | 1,249 | 19,747 | 590,391 | 6,538 | 79,938 |
| C3 | 35,089 | 108,966 | 1,747 | 273,706 | 2,366 | 35,113 | 523,719 | 43,348 | 117,819 |
| C4 | 40,264 | 103,724 | 1,198 | 269,154 | 12,426 | 58,942 | 584,556 | 8,463 | 64,472 |
| D1 | 198,866 | 220,854 | 32,035 | 270,721 | 10,407 | 86,644 | 330,131 | 26,019 | 138,600 |
| D2 | 235,850 | 184,124 | 17,841 | 288,731 | 3,474 | 62,469 | 361,768 | 10,709 | 117,998 |
| D3 | 189,675 | 230,154 | 22,685 | 282,294 | 2,679 | 79,248 | 329,751 | 27,660 | 158,560 |
| D4 | 195,073 | 225,096 | 17,052 | 274,952 | 12,534 | 110,355 | 366,190 | 7,475 | 117,699 |
| E1 | 2,159 | 9,869 | 125 | 260,350 | 29,000 | 24,599 | 608,074 | 65,820 | 79,965 |
| E2 | 2,009 | 10,004 | 40 | 291,579 | 15,007 | 7,780 | 678,141 | 21,624 | 38,815 |
| E3 | 1,921 | 10,066 | 105 | 287,545 | 9,192 | 23,637 | 598,400 | 82,360 | 77,876 |
| E4 | 2,056 | 9,927 | 63 | 253,713 | 43,295 | 25,728 | 670,295 | 17,321 | 44,752 |
| F1 | 692,539 | 135,380 | 65,157 | 253,893 | 51,233 | 119,715 | 8,078 | 2,834 | 4,575 |
| F2 | 731,292 | 96,294 | 30,987 | 284,960 | 26,341 | 89,374 | 9,248 | 1,902 | 4,176 |
| F3 | 728,788 | 99,081 | 32,509 | 282,855 | 26,439 | 90,463 | 7,609 | 3,313 | 5,861 |
| F4 | 695,059 | 132,666 | 61,666 | 252,708 | 50,796 | 116,988 | 9,454 | 1,573 | 6,381 |

Table 7. Order submission statistics in different experiment groups. Here ILO, GALO and ZILO represent the average limit order submission from the informed traders, GA traders and ZI traders respectively. Similarly, IMO, GAMO and IMO represent the average market orders of traders, while ILE, GALE and ZILE represent the average executive limit order for the informed traders, GA traders and ZI traders respectively. The results are based on 30 simulations.

5.1. **Order submission.** Clearly, the order submission behavior of traders depends on the information lag, information value, and order aggressiveness of traders. The total number of order submissions is positively related to the number of each type of traders. When the market becomes less informative, the informed traders submit more market orders than limit orders so that they can benefit from their short-lived private information. For the uninformed traders, when they become less aggressive and when the market becomes less informative, they tend to submit limit orders. The results presented in Tables 7, 8 and 9 provide supporting evidence on these intuitions. We also provide more details on the impact of learning and information-lived time on order submission in Table A.4 in the Appendix B. Based on 30 simulations for each experiment group, we report the means of the total number of the limit orders (LO) and market orders (MO) submitted, and the limit orders executed (LE) by the informed (denoted by ILO, IMO, and ILE in columns 2 to 4, respectively), GA (denoted by GALO, GAMO, and GALE in columns 5 to 7, respectively), and ZI (denoted by ZILO, ZIMO, and ZILE in columns 8 to 10, respectively) traders in Table 7. The results in Table 7 indicate the total number of orders submitted for different types of traders for the BM and four scenarios listed in Table 2. We also report the ratio of the total market order to the total limit order (MO/LO in column 2) the decompositions of the ratio to the ratios for the informed traders (IMO/LO in column 3), GA traders (GAMO/LO in column 4), and ZI traders (ZIMO/LO in column 5) and the market order fractions (columns 6-8) of three types of traders for different market structures in Table 8. Following Bloomfield et al. (2005), we use the submission and taking rates to analyze traders' order submission in different experiment groups. The *submission rate* of a trader, defined by the ratio of the numbers of the limit orders to the total orders submitted by the trader, measures liquidity supply; while the *taking rate*, defined by the ratio of the numbers of the market orders to total executed orders of the trader, measures liquidity consumption. We also use the *trading rate*, defined by the ratio of the numbers of the executed orders of the trader to the total executed orders of the market, to measure the trading activity and dominance of the trader in the market. The submission, taking, and trading rates are reported in Table 9, where ISub, GASub, and ZISub in columns 2 to 4 represent the submission rates, ITake, GATake, and ZITake in columns 5 to 7 represent the taking rates, and IT, GAT, and ZIT in columns 8 to 10 represent the trading rates of the informed, GA, and ZI traders, respectively.

| Experiment | MO/LO | IMO/LO | GAMO/LO | ZIMO/LO | IMO/MO | GAMO/MO | ZIMO/MO |
|---|---|---|---|---|---|---|---|
| BM | 19.11% | 12.82% | 1.51% | 4.79% | 67.07% | 7.88% | 25.04% |
| D1 | 32.17% | 27.62% | 1.30% | 3.25% | 85.84% | 4.05% | 10.11% |
| E1 | 12.03% | 1.13% | 3.33% | 7.56% | 9.43% | 27.70% | 62.87% |
| F1 | 19.85% | 14.18% | 5.37% | 0.30% | 71.46% | 27.04% | 1.50% |

TABLE 8. The ratios of total market order to total limit order (MO/LO), their fractions (IMO/LO, GAMO/LO, ZIMO/LO) and market order fractions (IMO/MO, GAMO/MO, ZIMO/MO) of the informed, GA and ZI traders in different market structures.

The results in Tables 7,8 and 9 implies that

(v) *the informed traders mainly consume liquidity and submit order much more aggressively than the uninformed traders, while the uninformed traders mainly supply the liquidity in the normal market (but submit more market orders in the two extreme market structures comparing to the normal market structures).*

In fact, for the market orders and the trading rate, we compare the order submission with the same order aggressiveness for all traders in all market structures. Firstly, Table 8 indicates that, except for the market structure E with only one informed trader, the informed traders submit much more market orders than the uninformed traders (with the fraction of 67.07% for case BM, 85.84% for D1 and 71.46% for F1). Secondly, the taking rates in Table 9 for the informed traders are much higher than the uninformed traders, meaning that the most executed orders from the informed traders are market orders. Thirdly, in scenarios C, D and E in Table 9, the trading rate of the informed traders are higher than their market proportion. As the aggressive limit order is more likely to be executed, this implies that the informed traders submit orders more aggressively. This result is consistent with Menkhoff et al. (2010) who use trading activity as a proxy to identify the informed and uninformed traders. The result shows that informed traders mainly consume the liquidity in the normal market. Lastly, there are much higher limit order submissions for the GA and ZI traders (indicated by GALO and ZILO) than for the informed traders (measured by ILO) in Table 7 and higher submission rates from GA and ZI traders (measured by GASub and ZISub) than their taking rates (measured by GAT and ZIT) in Table 9. Hence the uninformed traders supply the liquidity. With the fixed number of the GA traders, Table 8 indicates that the GA traders submit more market orders in the two extreme market structures E and F than the normal market conditions C and D. Also, the taking rates for the uninformed traders (indicated by GATake and ZITake) in Table 9 are higher in the two extreme scenarios E and F than in the normal market scenarios BM and D. Therefore the uninformed traders submit more market orders in the two extreme market structures.

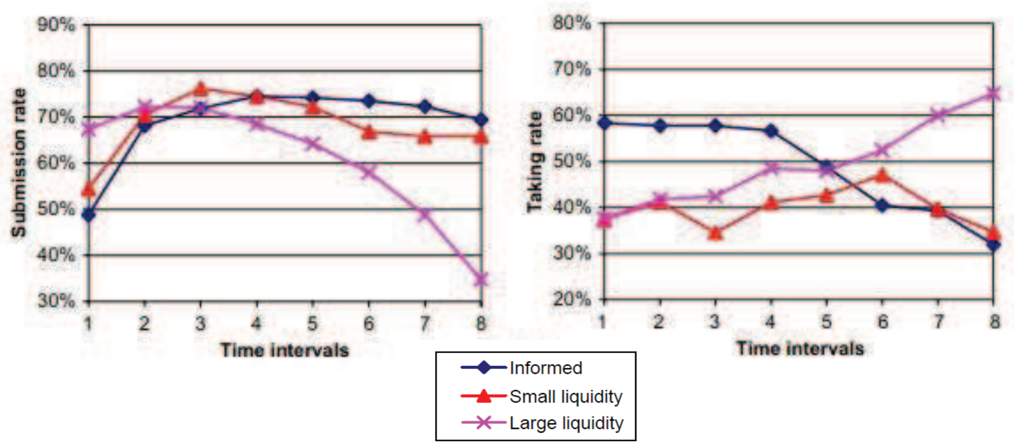
The order submission depends on information value and high information value leads to more market orders. The results show similar trading patterns observed in empirical studies. In a human subject based experiment market, Bloomfield et al. (2005) find that, informed traders prefer to submit market orders when their information value is high (e.g. in the early period of a trading day), but to submit limit orders when their information value is low (e.g. towards the end of a trading day). Therefore, their submission rate rises while taking rate declines during the trading day. However, for uninformed trader, their submission rate declines whereas the taking rate rises over the day. This result is illustrated in Figure 2(a) reported in Bloomfield et al. (2005). The same intraday trading patterns have been found

| Experiment | ISub | GASub | ZISub | ITake | GATake | ZITake | IT | GAT | ZIT |
|---|---|---|---|---|---|---|---|---|---|
| BM | 26.57% | 95.43% | 93.03% | 97.62% | 24.36% | 25.31% | 34.35% | 16.18% | 49.47% |
| C2 | 35.33% | 99.56% | 98.90% | 98.66% | 5.95% | 7.56% | 46.77% | 10.40% | 42.83% |
| C3 | 24.36% | 99.14% | 92.36% | 98.42% | 6.31% | 26.90% | 35.79% | 12.12% | 52.10% |
| C4 | 27.96% | 95.59% | 98.57% | 98.86% | 17.41% | 11.60% | 42.10% | 28.64% | 29.26% |
| D1 | 47.38% | 96.30% | 92.69% | 87.33% | 10.72% | 15.81% | 49.15% | 18.86% | 31.99% |
| D2 | 56.16% | 98.81% | 97.12% | 91.17% | 5.27% | 8.32% | 50.92% | 16.63% | 32.45% |
| D3 | 45.18% | 99.06% | 92.26% | 91.03% | 3.27% | 14.85% | 48.53% | 15.73% | 35.74% |
| D4 | 46.43% | 95.64% | 98.00% | 92.96% | 10.20% | 5.97% | 49.40% | 25.07% | 25.53% |
| E1 | 17.95% | 89.98% | 90.23% | 98.75% | 54.11% | 45.15% | 4.77% | 25.60% | 69.63% |
| E2 | 16.72% | 95.11% | 96.91% | 99.60% | 65.86% | 35.78% | 10.77% | 24.43% | 64.80% |
| E3 | 16.03% | 96.90% | 87.90% | 98.97% | 28.00% | 51.40% | 5.00% | 16.15% | 78.84% |
| E4 | 17.16% | 85.42% | 97.48% | 99.37% | 62.73% | 27.90% | 7.08% | 48.92% | 44.00% |
| F1 | 83.65% | 83.21% | 74.03% | 67.51% | 29.97% | 38.25% | 52.93% | 45.12% | 1.96% |
| F2 | 88.36% | 91.54% | 82.94% | 75.65% | 22.76% | 31.29% | 51.10% | 46.46% | 2.44% |
| F3 | 88.03% | 91.45% | 69.67% | 75.30% | 22.62% | 36.11% | 51.07% | 45.37% | 3.56% |
| F4 | 83.97% | 83.26% | 85.74% | 68.27% | 30.27% | 19.78% | 52.51% | 45.34% | 2.15% |

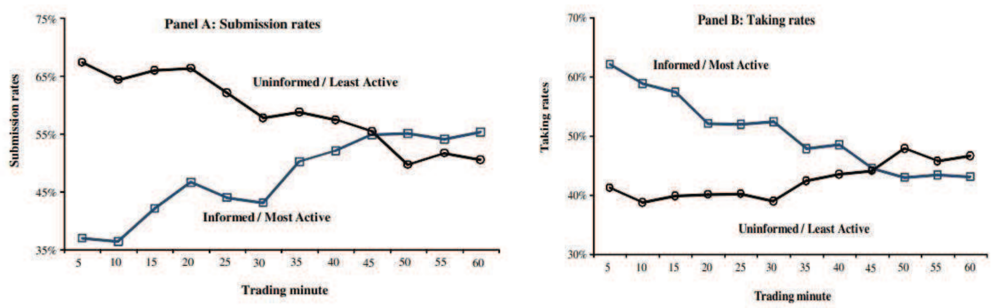
TABLE 9. The submission, taking and trading rates based on 30 simulations for each experiment group. Here ISub, GASub, and ZISub represent submission rates, ITake, GATake, and ZITake represent taking rates, and IT, GAT, and ZIT represent trading rates of the informed, GA and ZI traders respectively.

in Menkhoff et al. (2010) for the Moscow Interbank Currency Exchange(MICE), which are illustrated in Figure 2(b). In our experiments, the information value can be measured by the number of the informed traders in different market structure. In Group E, the proportion of the informed trader is low, hence the information value is high. As the proportion of the informed trader increases, the information value declines. For example, the information value is the highest in experiment E1, followed by BM, D1, and then F1. We plot the submission and taking rates of the four experiments in Figure 2(c) with respect to the increase in the fraction of the informed traders. If we interpret such change in the market structure as the change in the information value (from the highest to the lowest) in a trading day, Figure 2(c) shows similar patterns to Bloomfield et al. (2005) and Menkhoff et al. (2010). Therefore,

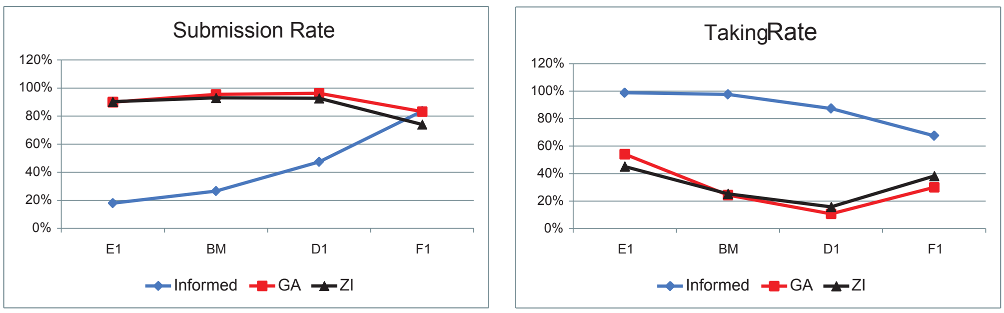
(vi) *when the proportion of the informed traders is low or the information value is high, the informed traders submit more market orders while the uninformed traders submit more limit orders. However, as the proportion of the informed trader increases or the information value declines, the informed traders submit more limit orders while the uninformed traders submit more market orders.*




(a) Bloomfield et al. (2005)




(b) Menkhoff et al. (2010)




(c) The model

FIGURE 2. The order submission patterns of (a) Bloomfield et al. (2005) model, (b) Menkhoff et al. (2010) model, and (c) the model of this paper.

The reduction of the uninformed traders' order aggressiveness increases the limit orders, as showed by GALO and ZILO in Table 7. When the uninformed traders reduce their order aggressiveness, they submit more limit orders that would improve the liquidity. However, the executed limit orders do not increase but decline (see GALE and ZILE in Table 7). This implies that the uninformed traders actually submit less-aggressive limit orders, thus the market liquidity declines. Therefore,

(vii) *the reduction of the uninformed traders' order aggressiveness increases the limit orders but reduces the market liquidity.*

This result is also confirmed be the bid-ask spread discussed in the following. This finding is consistent with Menkhoff et al. (2010) who find that the uninformed traders generally treat aggressive limit orders as patient (less-aggressive) limit orders. Hence the reduction of the order aggressiveness of the uninformed traders also widens the bid-ask spread (see more discussion in the following).

5.2. **Bid-ask spread.** Bid-ask spread is an important measure of information dissemination efficiency. Early studies, such as Glosten and Milgrom (1985), Kyle (1985), and Easley and O'Hara (1987), use asymmetric information to explain the formation of bid-ask spread in market maker markets. In limit order markets, Glosten (1994) firstly shows that asymmetric information causes bid-ask spread. De Jong, Nijman and Roell (1996), Brockman and Chung (1999), and Ahn, Cai, Hamao and Ho (2002) empirically study the composition of bid-ask spread and find that information cost is an important source of bid-ask spread.

In the microstructure literature, bid-ask spread is closely related to the probability of informed trading (PIN). In our model, we can explicitly calculate the trading rate of the informed traders (IT) as showed in Table 9, which can be used to represent the PIN. Clearly, the PIN is positively related to the market fraction of the informed traders. When information is long-lived, Rosu (2010) proposes the ratio of intraday price volatility to the average bid-ask spread as a measure for the PIN. He demonstrates that a higher fraction of informed traders generates smaller bid-ask spreads, meaning a negative relation between the spread and PIN. However, when the information is short-lived, it is not clear how the spread, volatility, and PIN are related. In our model, by comparing different scenarios, we are able to examine their relationship. We present the results on the bid-ask spread and its volatility in Table 10.[10] We also plot in Figure 3 the changes in the spread, volatility, together with the PIN, with respect to the increase in the number of the informed traders. Based on the information of the informed trading, we can estimate the PIN directly by the trading rate of the informed traders presented in column IT in Table 9. Based on Table 10 and Figure 3, we have the following two observations.

[10]In the BM scenario, the bid-ask spread is 2.9 tick sizes which is close to the Shanghai Stock Market in China. The 2012 Market Quality Report of Shanghai Stock Exchange reports the average spread of 2.1 from 2005 to 2011, and 2.9 in 2008, which is close to the spread in the BM case in Table 10.

| Experiment | Spread | Variance | Experiment | Spread | Variance |
|---|---|---|---|---|---|
| BM | 2.9 | 4.0 | D1 | 4.6 | 13.0 |
| C2 | 6.4 | 14.0 | D2 | 9.2 | 26.0 |
| C3 | 2.8 | 3.0 | D3 | 5.3 | 17.0 |
| C4 | 3.2 | 4.0 | D4 | 5.3 | 14.0 |
| E1 | 1.6 | 1.2 | F1 | 9.6 | 67.0 |
| E2 | 1.8 | 1.4 | F2 | 14.2 | 87.0 |
| E3 | 1.5 | 1.0 | F3 | 12.7 | 98.0 |
| E4 | 1.4 | 0.6 | F4 | 9.7 | 73.0 |

Table 10. The bid-ask spreads in tick sizes and its volatility for different scenarios based on 30 simulations.

(viii) *The bid-ask spreads become wide when the uninformed traders reduce their order aggressiveness, which in turn reduce the market orders and increase the limit orders of the informed traders.*

For example, in both Groups C and D, the spreads are about 2.9 (for BM case) or 4.6 (for D1) ticks when the uninformed traders are more aggressive, but become doubled (6.4 for C2 or 9.2 for D2) when they become less aggressive. This effect becomes less significant when the market is less informative (the bid-ask spread changes from 1.6 to 1.8 for Group E), but becomes highly significant when the market becomes more informative (the bid-ask spread increases from 9.6 in F1 to 14.2 in F2). As we have discussed in the previous subsection, when the uninformed traders become less aggressive, they tend to submit more less-aggressive limit orders, which widen the spreads and reduce the market liquidity. When the fraction of informed trader increases, the informed traders consume more liquidity and the increase in the spread becomes more significant. Combining Tables 7, 9 and 10, we can see clearly that when the bid-ask spread becomes wide, the informed traders reduce their market orders and increase their limit orders (see IMO and ILO in Table 7). This observation is consistent with Menkhoff et al. (2010) who find that informed traders are very sensitive to bid-ask spread. Recalling the order submission rule in Table 1, a trader submits market order if his expected fundamental value $p^i_{t'}$ is more than $a_{t'} + \mu$ or less than $b_{t'} - \mu$, while submits limit order if $p^i_{t'} \in [b_{t'} - \mu, a_{t'} + \mu]$. Note that the expected fundamental value $p^i_{t'}$ of the informed traders is exactly equal to the fundamental value. Hence if the market price is not far away from the fundamental value, with a wide bid-ask spread, the probability that $p^i_{t'}$ for the informed trader lies in $[b_{t'} - \mu, a_{t'} + \mu]$ increases and consequently the informed traders submit more limit orders and reduce market orders. Hence, when the reduction of the order aggressiveness of the uninformed traders widens the bid-ask spread, the informed traders reduce their market orders and increase their limit orders. This result shows that the informed and uninformed traders indirectly interact via bid-ask spread.

(ix) *The bid-ask spread and its volatility, and the PIN are positively related to the fraction of the informed traders.*

This is illustrated in Figure 3, which is different from Rosu (2010) who demonstrates a negative relation between the spread and the PIN. This difference shows that learning and the live-time of information can affect the spread differently. We now provide an explanation to this difference. In Rosu (2010), information-lived time is long. Hence the informed traders have less competition with each other and they prefer to use limit order to increase the profit opportunity. When the fraction of informed traders is high, they tend to submit more aggressive limit orders, which narrow the bid-ask spread. Therefore the PIN and the bid-ask spread is negatively related. In contrast, when information-lived time is short, the learning from the GA traders improves the market information dissemination efficiency (shown in the previous section). Therefore the informed traders prefer to use market order in order to benefit from their information within limited time horizon. Based on the results in Table 8, when the fraction of the informed traders increases (from 1 in E1 to 12 in BM and then to 35 in D1), the fraction of the market orders to the limit orders $MO/LO$ increases and the increase is mainly due to the informed traders (comparing $IMO/LO$ to $GAMO/LO$ and $ZIMO/LO$). Consequently the informed traders consume the market liquidity and widen the bid-ask spread. Therefore the PIN and the bid-ask spread and its volatility is positively related as showed in Figure 3. In the extreme case F, we observe the largest bid-ask spread in Table 10 but a reduced $MO/LO$ and $IMO/LO$ in Table 8. In this case, there is only one ZI trader. Hence the liquidity supply from the ZI trader decreases dramatically. Due to the high proportion of the informed trader, they submit more market orders (indicated by high $IMO/MO$ in Table 8). This widens the bid-ask spread dramatically. Therefore the informed traders reduce their market orders, which reduce the $MO/LO$ and $IMO/LO$ (comparing to case D1) in Table 8. Note that when the information-lived time becomes very long, the bid-ask spread increases, which is illustrated in Table A.5 of Appendix B. When the information lived time changes from 60 to 240 in experiments B1 and B2, the bid-ask spread does not change significantly. However, when the information-lived time becomes very long ($\tau = 1200$) in experiment B3, the bid-ask spread increases significantly. This indicates that the difference of observation (ix) from Rosu (2010) is mainly due to the learning of the uninformed traders when the information is short-lived.

In addition, the implication of our results to the effect of earning announcements is also different from Rosu (2010). To explain the large spreads around earning announcements, Rosu (2010) conjectures that the widen spread is not caused by asymmetric information but either lower trading activity or larger fundamental volatility. Ruchti (2012) finds empirically that the bid-ask spread and asymmetric information[11] are high before earning announcement and low after earning announcement. Our results are consistent with this empirical observation. In fact, before earning announcement, asymmetric information is larger, so the information value is higher, which attracts more informed traders to use more market order. Consequently, the informed traders consume market liquidity, widening the bid ask spread. Similarly, the spread becomes small after the announcement.

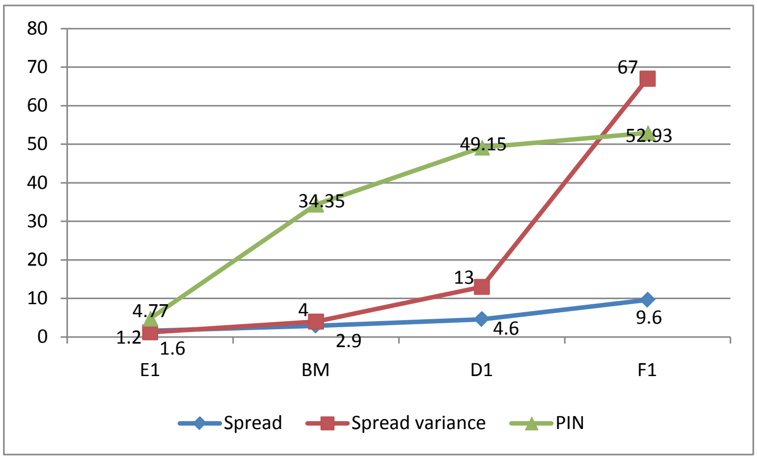



FIGURE 3. The change in the bid-ask spread, variance and probability of informed trading (PIN) wit respect to the increase in the fraction of the informed traders. The bid-ask spread and its variance are measured by tick sizes, while the PIN is measured by the trading rate of the informed traders.

5.3. **Order profit.** Intuitively, the informed traders gain from the uninformed traders due to the information value and the GA traders should perform better than the ZI traders due to the learning. Also, the profit opportunity of the traders decreases when they become less aggressive. These intuitions are confirmed by the average order profits presented in Table 11 and Figure 4. More precisely,

[11]Ruchti (2012) uses informedness ratio to measure asymmetric information, a higher value of informedness ratio may be indicative of large asymmetric information or a preponderance of information trading, while a lower value indicates more liquidity motivate trading and hence, less asymmetry information.

(x) *the profit of the informed traders increases when the information becomes more valuable, and the informed traders profit from the uninformed traders while the GA traders perform better than the ZI traders.*

This is demonstrated by a comparison between scenarios E1, A1, D1 and F1 where the information value decreases as the number of the informed traders increases. In addition, as the information lag increases as in B1, BM, B2, and B3, the profit of the informed traders increases significantly. The GA and ZI traders make loss to the informed traders and the GA traders perform better than the ZI traders. Note that for three exceptional scenarios E1, B2 and B3, the GA traders can make profit from the ZI traders due to their learning. In E1, there is only one informed trader and the market is less informative. In B2 and B3, the relative information advantage of the GA traders improved by using more historical prices. This shows that the GA traders are able to learn and perform better than ZI traders.

| Exp. | $r_I$ | $r_{GA}$ | $r_{ZI}$ | $R_I$ | $R_{GA}$ | $R_{ZI}$ |
|---|---|---|---|---|---|---|
| A1 | 0.1858 | None | -0.0925 | 20,796 | None | -20,796 |
| A2 | 0.1679 | -0.1145 | None | 17,235 | -17,235 | None |
| A2_Extra | 0.0898 | -0.082 | None | 6,128 | -6,128 | None |
| BM | 0.1786 | -0.0099 | -0.1208 | 19,360 | -481 | -18,879 |
| B1 | 0.1284 | -0.0246 | -0.11 | 11,930 | -609 | -11,321 |
| B2 | 0.2427 | 0.0033 | -0.1288 | 29,261 | 406 | -29,667 |
| B3 | 0.5224 | 0.1495 | -0.2196 | 68,079 | 30,739 | -98,818 |
| D1 | 0.1371 | -0.1162 | -0.142 | 34,777 | -11,320 | -23,457 |
| E1 | 0.1965 | 0.1187 | -0.0572 | 1,967 | 6,479 | -8,445 |
| F1 | 0.0843 | -0.0944 | -0.1034 | 16,929 | -16,162 | -767 |

TABLE 11. The order profits for some experiments. Here $R_I$, $R_{GA}$ and $R_{ZI}$ represent the total order profit and $r_I$, $r_{GA}$ and $r_{ZI}$ represent the average profit per order for the informed, GA, and ZI traders respectively. In A2_Extra, the information lived-time $\tau$ is 30.

One may argue that, if the uninformed traders always lose, why do they still want to participate in the market? We answer this question from two aspects. First our model is a zero-sum game which focuses on the order profit of each transaction, instead of the whole investing period. In real markets, some traders acting as ZI traders may trade due to their liquidity demand or dividend payment, so they can tolerate losses in some transactions. Secondly, the GA traders can make profit when the proportion of the informed traders is low and the proportion of ZI traders is high, or when the information-lived time is very long. More importantly, when information is costly, the informed traders need to pay the cost for information acquisition. When the information-lived time is very short, the difference of the order profit between the informed and the GA traders becomes smaller. This is confirmed in an extra experiment A2_Extra in which the market structure is the same as in experiment A2 but the information-lived time $\tau$ is shorted to 30. Comparing experiment A2_Extra with A2, the order profit becomes significantly higher for the GA traders and lower for the informed traders. Therefore, if we take into account of the information cost for the informed traders, the GA traders may not always lose.

The order aggressiveness has different impact on different traders. For the informed traders, the impact of order aggressiveness of the uninformed traders is less significant when the market information value is extremely high or low (Scenarios E and F), but highly significant in normal market conditions (Scenarios C and D), which is illustrated in the upper panel of Figure 4. In addition, the profit for the informed traders reduces when all the uninformed traders become less aggressive. When the uninformed traders become less aggressive, their liquidity supply reduces, which limits the profit opportunity of the informed traders. For the uninformed traders, the impact of the order aggressiveness on their order profits is significant when the information value is lower (Scenarios C and E), as illustrated in the middle and low panels of Figure 4. In particular, in Scenario E when information value is lowest, the GA traders has great information advantage than the ZI traders. Hence the profit opportunity for the GA traders increases (decreases) when they become more (less) aggressive. Therefore,

(xi) *the order aggressiveness of the uninformed traders is positively related to the profit opportunity of the informed traders in normal market; however, lower information value provides the GA traders opportunity to profit from the ZI traders when they become more aggressive.*

Hence, if the uninformed traders can determine their order aggressiveness endogenously, then some smart uninformed traders such as the GA traders are likely to become more (less) aggressive when the market information value is higher (lower).

## 6. Conclusion

It is very important and challenging to understand information-based trading in financial markets and in particular how uninformed traders learn from market information and behave when they trade with informed traders, and how market efficiency is affected under learning. This paper presents an artificial stock market model of continuous double auction with both informed and uninformed traders to tackle these issues. When information is short-lived, the uninformed traders can learn from the lagged fundamental values and market information through genetic algorithm. By examining different scenarios, we show that learning, together with information-lived time, information value, and order aggressiveness, can have significant impact on market information dissemination efficiency, bid-ask spread of limit order books, order submission behavior, and order profit of informed and uninformed traders.

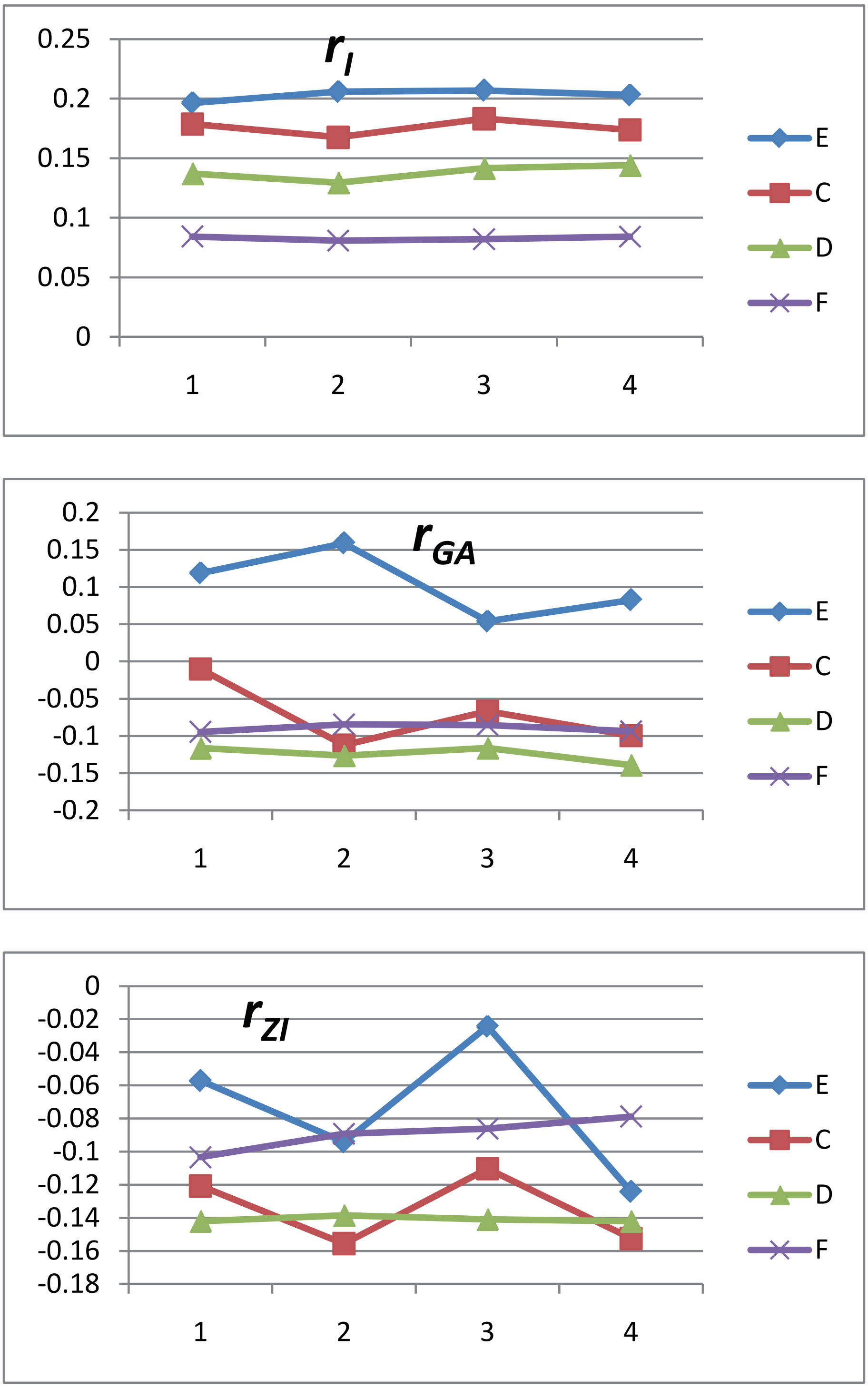



FIGURE 4. Order aggressiveness and order profit. 1,2,3,4 are experiment numbers in each experiment group, for example, 1 means C1 in Group C.

Consistent with learning literature, we find that, when information is short-lived, learning improves market efficiency of information dissemination and the information acquisition of the uninformed traders becomes more effective. Also, under the learning, the informed traders mainly consume market liquidity and submit aggressive orders, while the uninformed traders mainly supply market liquidity. More interestingly, different from the literature with long-lived information, we find that the bid-ask spread, its volatility, and the PIN are positively related. Therefore the informed and uninformed traders interact not only via market prices but also via bid-ask spreads. In general, the informed traders gain from the uninformed traders due to the information value and the GA traders perform better than ZI traders due to the learning.

Overall, we show that not only the rational behavior of the informed traders but also the learning of the uninformed traders can affect the price formation, dynamics of the limit order book, and traders' behavior. The model proposed in this paper provides some insights into some limit order book phenomena documented in empirical literature. It helps us to better understand the dynamics of limit order markets when traders are learning, which is difficult to model in the traditional microstructure literature. In particular, the model characterizes the interaction between informed and uninformed traders and its impact on the market.

The model in this paper can be developed further to model more complicated learning and adaptive behavior of traders. In this paper, when learning from the market information, the uninformed traders use only the average market prices and mid-prices. More realistically, they may also learn from other market information such as the types of the last trading, trading volume, the depth and shape of the order book. Also, based on the performance and information cost, traders may switch between informed and uninformed strategies. Whether more complicated learning and adaptive behavior of traders improve information dissemination efficiency and profit opportunity of traders is an interesting issue to be explored in future research.

## Appendix

### A. Further analysis on the information dissemination efficiency.

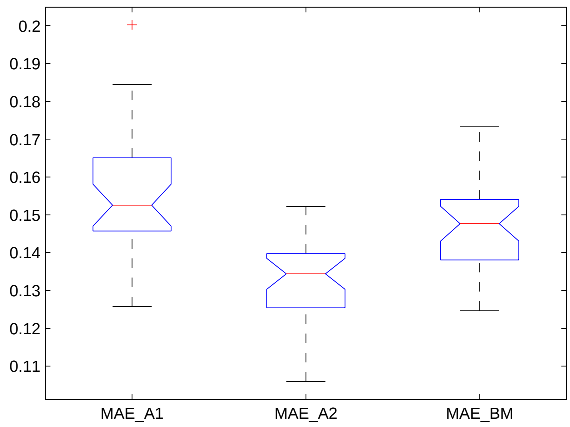



Figure A.1. ANOVA analysis of scenario A.

| Experiment | Investor Proportion | $\omega_{GA}$ | $\omega_{ZI}$ | $MAE_p$ | $MAE_{pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|---|---|
| D1 | 35:30:35 | 0 | 0 | 0.1079 | 0.1364 | 0. 60% | 0.1413 | 0.1512 |
| D2 | 35:30:35 | 0.04 | 0.04 | 0.0874 | 0.1275 | 0. 45% | 0.1303 | 0.1426 |
| D3 | 35:30:35 | 0.04 | 0 | 0.1069 | 0.1345 | 0. 60% | 0.1405 | 0.1518 |
| D4 | 35:30:35 | 0 | 0.04 | 0.1036 | 0.1306 | 0. 53% | 0.1405 | 0.1507 |

Table A.1. The effect of order aggressiveness in experiment group D.

| Experiment | Investor Proportion | $\omega_{GA}$ | $\omega_{ZI}$ | $MAE_p$ | $MAE_{pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|---|---|
| E1 | 1:30:69 | 0 | 0 | 0.1719 | 0.1738 | 0. 93% | 0.1659 | 0.1778 |
| E2 | 1:30:69 | 0.04 | 0.04 | 0.1785 | 0.1841 | 0. 94% | 0.1710 | 0.1849 |
| E3 | 1:30:69 | 0.04 | 0 | 0.1837 | 0.1856 | 0. 98% | 0.1732 | 0.1865 |
| E4 | 1:30:69 | 0 | 0.04 | 0.1759 | 0.1787 | 0. 99% | 0.1718 | 0.1839 |

Table A.2. The effect of order aggressiveness in experiment group E.

| Experiment | Investor Proportion | $\omega_{GA}$ | $\omega_{ZI}$ | $MAE_p$ | $MAE_{pm}$ | $MRE$ | $MAD_{GA}$ | $MAD_{ZI}$ |
|---|---|---|---|---|---|---|---|---|
| F1 | 69:30:1 | 0 | 0 | 0.0636 | 0.0995 | 0. 32% | 0.1046 | 0.1271 |
| F2 | 69:30:1 | 0.04 | 0.04 | 0.0563 | 0.0973 | 0. 34% | 0.0923 | 0.1195 |
| F3 | 69:30:1 | 0.04 | 0 | 0.0628 | 0.0973 | 0. 34% | 0.1026 | 0.1241 |
| F4 | 69:30:1 | 0 | 0.04 | 0.0553 | 0.096 | 0. 31% | 0.0945 | 0.1227 |

Table A.3. The effect of order aggressiveness in experiment group F.

## B. Further analysis on the order submission and bid-ask spread.

| Experiment | ILO | IMO | ILE | GALO | GAMO | GALE | ZILO | ZIMO | ZILE |
|---|---|---|---|---|---|---|---|---|---|
| A1 | 34,651 | 109,288 | 2,379 | None | None | None | 795,584 | 58,777 | 165,687 |
| A2 | 43,744 | 100,227 | 2,216 | 787,773 | 26,346 | 124,356 | None | None | None |
| B1 | 53,676 | 90,331 | 2,450 | 248,128 | 3,359 | 22,297 | 526,025 | 16,869 | 85,812 |
| B2 | 26,783 | 117,307 | 3,109 | 262,427 | 26,505 | 58,173 | 516,599 | 73,352 | 155,882 |
| B3 | 22,323 | 121,676 | 8,460 | 238,942 | 77,886 | 119,186 | 447,970 | 185,947 | 257,863 |

Table A.4. Order submission statistics for experiment Group A and B. Here ILO, GALO and ZILO represent the average limit order submission from informed traders, GA traders and ZI traders respectively. Similarly, IMO, GAMO and IMO represent the average market orders of traders, while ILE, GALE and ZILE represent the average executive limit order for traders. The results are based on 30 simulations.

| Experiment | Spread | Variance |
|---|---|---|
| A1 | 2.8 | 4.0 |
| A2 | 3.1 | 4.0 |
| B1 | 3.1 | 4.0 |
| B2 | 3.1 | 6.0 |
| B3 | 7.4 | 111.0 |

Table A.5. The bid-ask spread in tick sizes in experiment Group A and B.

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