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Abstract

This paper examines the dynamics of financial distress and in particular the mechanism of transmission of shocks from the financial sector to the real economy. The analysis is performed by representing the linkages between microeconomic financial variables and the aggregate performance of the economy by means of a microfounded model with firms that have heterogeneous capital structures. The model is solved both numerically and analytically, by means of a stochastic approximation that is able to replicate quite well the numerical solution. These methodologies, by overcoming the restrictions imposed by the traditional microfounded approach, enable us to provide some insights into the stabilization policies which may be effective in a financially fragile system.

Keywords: Financial fragility, complex dynamics, stochastic aggregation.

JEL classification: E12, E22, E44

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1 Introduction

Minsky (1977) defines financial fragility as "*...an attribute of the financial system. In a fragile financial system continued normal functioning can be disrupted by some not unusual event*". The two key points highlighted by this definition are the "*not unusual event*" that may stop the normal functioning of a financial system and that the system in question must display a certain degree of *fragility*. As regards the former point, there is no shortage of interpretations in this sense of the crises that, at progressively shorter intervals, have hit the capitalist economies (Kindleberger, 2005; Reinhart and Rogoff, 2009). The idea of an intrinsic instability of the capitalist financial system dates back to Minsky (1963) and has gained increasing attention, especially during the recent financial crisis[1]. As regards the second point, the identification of the degree of systemic fragility, according to Minsky, involves a micro-level analysis, as it depends on the ratio of financially sound to financially distressed firms in the economy. More precisely, in his famous 1963 essay, Minsky classifies firms into hedge, speculative or Ponzi type. The first are the sound firms that can repay their debt and the interest on it. The second type are the ones able to meet only the interest due on outstanding debt while, for the Ponzi firms, their cash flow is insufficient to fulfil neither the repayment of capital nor the interest due on outstanding debts.

As Taylor and O'Connell (1985) point out "*Shifts of firms among classes as the economy evolves in historical time underlie much of its cyclical behavior. This detail is rich and illuminating but beyond the reach of mere algebra. What can perhaps be formalized are purely macroeconomic aspects of Minsky's theories.*". According to them, this is one of the reasons for which Minksy's work has been so far largely neglected.

Such is no longer the case. In recent years a consistent stream of research has started to deal with the microfoundation of macroeconomics with heterogeneous and evolving agents. Significant results in terms of replication of empirical stylized facts have been attained through the numerical solution of agent based models[2]. From an analytical perspective, the most relevant contribution has been provided by Aoki[3], whose framework seems to allow a comprehensive analytical development of Minsky's theory that satisfactorily encompasses its essential microeconomic foundation. Aoki adopts analytical tools originally developed in statistical mechanics. In his view, as the economy is populated by a very large number of dissimilar agents, we cannot know which agent is in which condition at a given time and whether an agent will change its condition, but we can know the present probability of a given state of the world. This approach hence focuses in particular on the evolution of agents' characteristics through time. The basic idea consists in introducing a meso-level of aggrega-

[1]See the working paper series of the Levy Economics Institute of Bard College: http://www.levy.org/vtype.aspx?doctype=13.

[2]See by way of example Axtell et al. (1996); Delli Gatti et al. (2005).

[3]Namely, Aoki (1996, 2002); Aoki and Yoshikawa (2006), with a further development provided by Di Guilmi (2008)

tion, obtained by grouping the agents into clusters according to a measurable variable. The dynamics of the number of firms in each cluster defines as well the evolution of the whole economy, which is identifiable by specifying some general assumptions on the stochastic evolution of these quantities. For example, assuming their dynamics to be a Markov process, it is possible to describe the stochastic evolution of these occupation numbers using the master equation, which is a standard tool developed in statistical mechanics to model the evolution of ensembles of particles. Interaction among agents is modelled by means of the *mean-field approximation* (Aoki, 1996) that, basically, consists in reducing the vector of observations of a variable over a population to a single value. The usefulness and the potential of this approach for analysing Minsky's theoretical structure appears to be promising.

The aim of this paper is to propose a financial fragility model, along the lines of Minsky (1975) and Taylor and O'Connell (1985), with heterogeneous and interacting firms, using first a numerical simulation of the agent based model and then comparing this solution with the one obtained by means of the stochastic dynamic aggregation technique mentioned above. Besides the technical contribution, such a framework should allow a deeper insight into the mechanism by which shocks are transmitted from the financial sector to the real economy. This aspect, which is central in Minsky's approach, is not the main focus of Taylor and O'Connell (1985). In both studies the market valuation of shares may differ from the present value of capital, with the difference being absorbed by net worth. Given the substitutability of assets, a shift of investor preferences impacts on firms' net worth via a different evaluation of capital assets. Therefore, investor expectations of future profits influence, on the one hand, the prices of firms' equities on the stock market and, on the other hand, the current value of firms' assets. For example, if the market forecasts a rise in the demand for a certain product, there will be an increase in the evaluation of the machines that produce that good and a contemporaneous rise in the price of shares for the firms that sell them (Wray and Tymoigne, 2008). These two effects shape firms' decisions on investment and, as a consequence, output and employment levels. At the aggregate level then the economy may experience periods of growth, depression or fluctuations due solely to changes in the market mood and not to its actual productivity. This mechanism was first studied by Keynes (1936) and has been subsequently modelled by Tobin (1969), Kalecki (1971) and the cited work of Minsky (1975). Taylor and O'Connell (1985) introduce into the original analysis of Minsky an exogenous variable which expresses the level of confidence of the market, isolating the effect of investors' expectations on the value of a firm's assets.

Our contribution concerns three main modifications that we bring to this original framework. The most relevant is the microfoundation of the financial fragility approach. As already stressed, the presence of heterogeneous agents and the consequent issues in consistently microfounding the framework are among the factors that have limited the diffusion through the economics profession of Minsky's approach. Here, differently from the original models, the equations are expressed with reference to the micro-level. We then study the macro-dynamics using two different approaches: the first being an agent based one, with the highest degree of heterogeneity, and the second being a stochastic approximation, obtained by means of Aoki's aggregation tools. The second modification derives from the observation that, from our perspective, the evaluation of capital assets comes from the stock market, in which investors display heterogeneous expectations about firms' future profits. Thus we link the new variable introduced by Taylor and O'Connell (1985) to the predominant strategy in the stock market. The third novel aspect regards the modelling of endogenous money which, if considered, is not analytically defined in the previous studies. In the present treatment it is linked to the overall amount of financial assets rather than being a multiple of the monetary base as typically presented in literature[4]. In our opinion this view is more consistent with the Minsky's idea from two different perspectives. First, from a formal point of view, Minsky, in particular in his later writings[5], seems to connect the degree of liquidity of the system to a wider range of marketable paper, such as for example, securities and derivatives, than the typical monetary aggregates. As a consequence financial innovation, besides having the effect of transferring risk, influences the credit supply also by affecting the endogenous determination of the degree of liquidity in the economy. Second, from a theoretical point of view, by modelling endogenous money as endogenous wealth, the availability of credit is linked to the conditions of the financial system and, in particular, to the expectations of investors, providing a quantitative benchmark for the analytical representation of his idea of increased propensity to supply credit in periods of expansions (Minsky, 1963). In this way we define a mechanism of endogenous credit creation, a concept largely neglected by the literature[6].

The outline of the paper is as follows. Section 2 describes the general features of the model and outlines the basic structure of firms in the agent based framework. The behavioural hypotheses and equations are the same for the agent based set up and for the stochastic approximation; in the former they are referred to each single firm, without limitation on the endogenous heterogeneity, while the stochastic dynamics consider a representative firm for each cluster. Section 3 defines the hypotheses for investors and the capital market. Section 4 discusses the stochastic approximation to a high order heterogeneous model. Section 5 presents the outcomes of the simulations for the agent based model and contrasts them with the stochastic approximation results. Section 6 concludes with some discussion of the questions that can be addressed using the framework developed here.

2 Firms

This section presents the structure of the agent based model. Variables are written with the superscript j when they refer to a generic firm, while the mean

[4]For a review see Fontana (2003).
[5]As in Minsky (2008) about debt securities.
[6]Exceptions are Jarsulic (1989) and Keen (1995).

field values, that are introduced in section 3, are identified by the subscript $z = 1, 2$. Aggregate variables appear without any sub- or superscript. The model is set up in continuous time. The hypotheses of the model are listed below.

- Due to informational imperfections in capital markets (Myers and Majluf, 1984; Greenwald and Stiglitz, 1990), firms prefer to finance their investments I^j with retained earnings F^j and, only if they are not sufficient, by the emission of new equities E^j or with new debt D^j.

- Firms are classified into two groups, clustering together the speculative and Ponzi firms of the Minsky (1963) taxonomy. In order to ease the calculations, analogously to Lima and Meirelles (2007), the threshold level of debt is set to 0. Therefore, the classification defines as *speculative* (type 1) the firms that have to finance their investment with debt or new equity and as *hedge* (type 2) the firms that can finance their investments with retained profits and do not need external sources. Thus firms can be classified into two states, depending on whether or not they display a positive debt in their balance sheet:

  - state $z = 1$: $D^j(t) > 0$,
  - state $z = 2$: $D^j(t) = 0$.

  This classification is the base for the analytical solution of the model that is detailed in section 4 below.

- Every period the j-th firm targets an amount of investment I^j. The new level of capital then determines the demand for labour and the output. The investment is decided on the base of the shadow-price of capital P_k^j (Tobin, 1969; Minsky, 1975), so that:

$$I^j(t) = aP_k^j(t), \tag{1}$$

  where a is a parameter measuring the sensitivity of firms to the current value of capital assets and the shadow price P_k is specified below. This formulation recalls the one adopted by Delli Gatti et al. (1999), while the model of Taylor and O'Connell (1985), very much in line with Minsky (1975), takes into account the price differential between the shadow price and the price of furnishing new investment goods. Our choice in (1) is motivated by the fact that the solution adopted by Taylor and O'Connell (1985) would add a factor that might turn out to be too noisy for the identification of the effects of financial markets fluctuations on investment.

- The selling price of the final good is obtained by applying a mark-up τ on the direct production costs according to

$$P(t) = (1 + \tau)w(t)b, \tag{2}$$

where w is the nominal wage and b is the labour-output ratio. The parameters τ and b are assumed to be constant while the variable w is defined below. These quantities are equal for all firms.

- Firms produce a good that can be used either for consumption or investment. All salaries are consumed and the unit salary varies in each period in order to ensure the equilibrium in the product market. In particular it is equal to

$$w(t) = \frac{I(t)}{\tau b X(t)} \tag{3}$$

  where $X(t)$ is the total output.

- Assuming that the firms adopt a technology with constant coefficients, the amount of labour requested is residually determined once the optimal level of investment, and hence of capital, is quantified. The supply of labour is infinitely elastic. The production function for all firms is written as

$$X^j(t) = G(K^j(t), L^j(t)) \tag{4}$$

  with K and L representing, respectively, physical capital and labour.

- The rate of profit r^j is given by

$$r = r^j(t) = \frac{\tau}{1+\tau}\frac{X^j(t)}{K^j(t)} \ , \tag{5}$$

  which is equal across firms since they use the same technology. Assuming that the fixed coefficient technology is constant r does not change over time.

- P_k^j is determined according to

$$P_k^j(t) = \frac{(r + \rho^j(t))P(t)}{i(t)} \ , \tag{6}$$

  where i is the interest rate and ρ^j is the expected difference of return to capital for the firm j with respect to the minimum level r. The variable ρ is introduced by Taylor and O'Connell (1985) in their analysis of the original Minsky model in order to link investors' expectations to the investment decision[7]; it plays a decisive role in their treatment as well as in the present one. Here we consider it as a function of the prevailing strategy on the financial market. This quantity is therefore a key variable in the mechanism of transmission of shocks from the financial markets to the real economy. The mechanism by which the process occurs is fully detailed in section 3. Combining (1) with (6) we obtain

$$I^j(t) = a\left[\frac{(r + \rho^j(t))P(t)}{i(t)}\right] . \tag{7}$$

[7] In Taylor and O'Connell (1985) it is defined as the expected difference between the anticipated return to holding capital and the present profit rate.

- Firms finance the part of investment that cannot be covered with internal funds by a fraction $\phi \; i(t)$ of equities, where $\phi > 0$ is a parameter, and then the rest with debt, the dependence on the interest rate reflecting the fact that in periods with a high interest rate equities would be preferred. The price of the new capital goods is assumed to be equal to the final goods price P. The sum of retained profits is indicated by F^j. Thus, the variations of E^j and D^j at an instant of time are given by

$$dE^j(t) = \phi \; i(t) \left[\frac{P(t)I^j(t) - F^j(t)}{P_{e1}(t)} \right] dt \tag{8}$$

$$dD^j(t) = \left[1 - \phi \; i(t)\right] \left[P(t)I^j(t) - F^j(t)\right] dt \tag{9}$$

  where P_{e1} is the price of equities for speculative firms to be defined in the following section.

- The timeline of the whole process over successive time intervals is shown in figure 1. In the first stage market expectations determine the shadow price of capital and the desired level of investment. In the following unit of time firms implement the decision, modifying the capital stock and producing the final good. The product is then sold in the following period, giving rise to a profit (or loss).

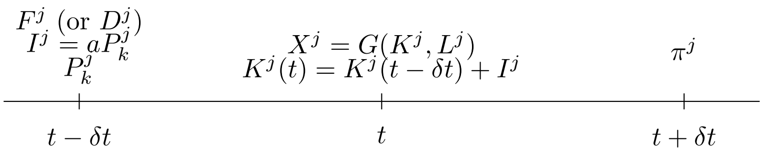

Figure 1: Timeline of the investment process.

- The balance sheet of a typical firm has the structure shown in Table 1. We use A to indicate the difference in the market valuations of assets and shares, less the eventual debt. Adopting the terminology of Taylor and O'Connell (1985) we term it as net worth. Actually, according to accounting conventions, retained profits are a component of firms' net worth and therefore they should be included into the latter. We indicate them separately in order to quantify the cash flow that can be used to finance future investment.

- Capital depreciates in each period at a constant rate v.

- Profits are given by

$$\pi^j(t) = P(t)X^j(t) - w(t)bX^j(t) - i(t)D^j(t) = \tau w(t)bX^j(t) - i(t)D^j(t). \tag{10}$$

Assets	Liabilities
	$P_e E^j$
$\frac{r+\rho}{i} K^j$	D^j (or F^j)
	A

Table 1: Structure of a generic firm's balance sheet

- Accordingly, the variation in retained profits, or cash flow, for a hedge firm is

$$dF^j(t) = \left[\pi^j(t) - P(t)I^j(t)\right] dt. \tag{11}$$

  If, at time t, $F^j(t) < P(t)I^j(t)$, the firm becomes speculative and $I^j(t) - F^j(t)$ will be financed with new equities and debt according to equations (8) and (9).

- A firm fails if its debt level exceeds some multiple of its capital stock, that is if

$$D^j(t) > c\, K^j(t) \tag{12}$$

  with $c > 1$. The probability of a new firm entering is directly proportional to the variation in the aggregate production with respect to the previous period. Thus in boom periods failed firms are rapidly replaced whilst in periods of distress this process can take quite a deal longer.

3 Investors

Even though a comprehensive modelling of stock markets would go beyond the aim of the present analysis, some behavioural assumptions on investors are needed for the internal consistency of the framework. This section illustrates the hypotheses and the conditions of equilibrium for the capital market.

3.1 Behavioural hypotheses

Investor preferences are modelled in a Keynesian fashion, assuming that a share of wealth is kept liquid. Minsky (1975) gives to the usual Keynesian motives for holding money (transaction, precautionary, speculative) a formal representation, modelling the demand for money as a function of income, interest rate, asset price, firm debt and near money supply. We model demand for money in a similar way. Moreover, we assume that the financial operators act according to a bounded rationality paradigm. Consequently, we classify them into the two broad categories of chartists and fundamentalists, within an approach that has an established tradition in the literature[8]. It has been demonstrated

[8] See for example Zeeman (1974); Chiarella and He (2003); Chiarella et al. (2009).

(Aoki and Yoshikawa, 2006, ch. 9) that this classification accounts for almost the totality of different possible strategies. We adopt this assumption that turns out to be particularly suitable in this framework. Indeed we can reasonably assume that, *on average*, fundamentalists, focusing on the real value of firms, will favour investment in hedge firms, while chartists, who base their decisions on extra-balance sheet information, may prefer riskier equities[9]. We assume that all investors maximize a CARA utility function in order to avoid the distinction between chartist and fundamentalist wealth.

Since our focus is on how changes in investor expectations impact on the real economy, we assume that variations in the proportion of the types of operators are not dependent on firms' performance and are simply governed by a stochastic law. This also allows for a wider range of possible outcomes and behaviours as a result of the multiplicity of exogenous factors (not related to the economy) that influence the markets.

3.2 The determination of ρ

As anticipated, the variable ρ plays a key role in the entire story, as it incorporates expectations that emerge in financial markets into the decision process of firms about investment. Taylor and O'Connell (1985) introduce it in order to better isolate the effect of the difference between the anticipated return and the current profit rate, an effect that in the original treatment of Minsky (1975) is directly incorporated in the shadow price P_k. As they were not interested in the impact of financial markets they did not explicitly model ρ, assuming independence between the behaviour of investors and firms. On the contrary since our perspective is mainly focused on the transmission of shocks from the financial sector, the role of ρ is reminiscent of Tobin's q (Tobin, 1969), in that it is connected to equity values. In this sense our work constitutes a bridge between these two approaches and indeed is an extension of them.

Two basic assumptions are at the root of the formulation of ρ: the first is its dependence on the relative proportion of chartists and fundamentalists in the market; the second concerns the formation of expectations. As anticipated, since fundamentalists look at the balance sheet of firms while chartists use other information and focus only on the evolution of returns, we can assume that an increase in the proportion of chartists heightens the expectations about indebted firms that, on the contrary, reduce when the share of fundamentalists is bigger. Accordingly, ρ^j is determined differently depending on whether a firm is in state 1 or in state 2, namely

$$\begin{aligned} \rho_1^j &= f_1(n^c) = \frac{n^c}{\tilde{\varpi}^j}, \\ \rho_2^j &= f_2(n^c) = \frac{1-n^c}{\tilde{\varpi}^j}, \end{aligned} \tag{13}$$

where $\tilde{\varpi}^j$ is an idiosyncratic random variable specified over a positive support.

[9]We adopt here an extensive definition of chartist strategy, referring to it as a generic alternative approach with respect to the fundamentalists rather than a pricing rule inferred by time series.

Since this random variable has the same support for each firm, on average a bigger fraction of chartists in the market leads firms in state 1 to increase their investments, their production and their debt. At the same time, the growing demand for credit puts pressure on interest rates. Therefore the system experiences a debt driven expansion that makes it progressively more vulnerable to sudden changes in investor expectations.

3.3 Equilibrium in the capital market

The equities of the two different types of firms can be correspondingly sorted into two classes with different associated risk[10]. Investors will allocate part of their wealth between the two classes of shares according to the market expectation at the time of choice. In order to make the pricing model analytically tractable, we follow the mean-field approach discussed in the introduction and replace all the interactions among the heterogeneous firms and the investors with an average interaction. Namely we make use of the two indicative values ρ_1 and ρ_2, calculated as a statistic of the ρ^j within each cluster of firms, which represent the benchmark values for investors in order to allocate their wealth and price the two types of shares.

The wealth W of investors is the sum of shares, bonds and money, so that

$$W(t) = P_{e1}(t)E_1(t) + P_{e2}(t)E_2(t) + D(t) + M(t). \tag{14}$$

where $M(t)$ is the nominal demand for money. Wealth evolves over time according to

$$\frac{dW}{dt} = \frac{dP_{e1}}{dt}E_1(t) + \frac{dP_{e2}}{dt}E_2(t) + P_{e1}\frac{dE_1}{dt} + P_{e2}\frac{dE_2}{dt} + \frac{dD}{dt} + \frac{dM}{dt}. \tag{15}$$

An initial endowment of money is assumed. Variations in total wealth are then due to capital gains, which in this framework constitute *high-powered wealth.*

Investors allocate their wealth among equities, firms' bonds and money according to the proportions: $\epsilon_1(i, \rho_1, \rho_2, \psi), \epsilon_2(i, \rho_1, \rho_2, \psi)$, $\beta(i, \rho_1, \rho_2, \psi)$ and $\Psi(i, \rho_1, \rho_2, \psi)$ that satisfy the constraint $\epsilon_1 + \epsilon_2 + \beta + \Psi = 1$. The parameter ψ reflects the propensity toward liquid assets and it is assumed to be constant over time. Given the structure of the equilibrium conditions, and in particular the fact that the demand for credit is always (partially) accommodated, the bigger is ψ the larger are M and the aggregate wealth W. Thus, this parameter may be interpreted as a proxy for the capacity of the system to generate endogenous credit[11]. The proportions of the two kinds of strategies influence ρ and through

[10] Actually, in each period, only speculative firms issue equities, given that hedge firms can finance all their investment with retained profits. Anyway in the market there are also the equities of firms that were speculative and became hedge, which would be assessed differently by investors.

[11] The introduction of ψ also allows us to provide a functional form for the demand of money which replicates the formulation of Minsky (1975, chap. 4). In his treatment it is given by the combined liquidity effects of the income Y, the interest rate r and the shadow price of capital P_k, the outstanding private financial commitments F and the supply of near money

this the allocation of wealth between the different assets. The equilibrium conditions on equities and credit markets are (time indices are omitted)

$$\begin{cases} \dfrac{\epsilon_1(i,\rho_1,\rho_2,\psi)}{P_{e,1}} W = E_1, \\ \dfrac{\epsilon_2(i,\rho_1,\rho_2,\psi)}{P_{e,2}} W = E_2, \\ \beta(i,\rho_1,\rho_2,\psi) W = D, \\ \Psi(i,\rho_1,\rho_2,\psi) W = M, \\ W = P_{e1}E_1 + P_{e2}E_2 + D + M. \end{cases} \tag{16}$$

The system (16) may be solved for the value of asset prices, interest rate, demand for money and aggregate rentiers' wealth. This latter turns out to be endogenously determined within the system in order to (partially) accommodate the demand for credit.

4 Stochastic dynamics

Our discussion so far has been in terms of single firms, referring all the variables to the agent level, and only in the last section have we introduced the mean-field approximations ρ_z. These variables allow us to set up the tools for the analytical solution of the model. Equations (1) and (6) can be computed starting from the mean-field values ρ_z in order to calculate the variables I_z and then to identify, using the other equations of the agent based model, two firms that are representative for each group. Next, studying the dynamics of the proportion of firms in each state, it is possible to obtain a complete analytical description of the system's dynamics. Therefore, the model is able to generate dynamics in two different ways: an *agent based* approach with N different agents and a *stochastic approximation*, with two different firms: one "good" and one "stressed". Two additional hypothesis are needed in order to develop the analytical solution: first, that firms switch from one state to another according to a jump Markov process; second, that the number of firms N is constant. The first step for this analytical treatment is the definition of probabilities at the micro level, such as the probabilities for a single firm to change its state.

4.1 Transition probabilities

The probability for a firm to transition from state 2 to state 1 depends upon its level of investment and retained profits. A hedge firm becomes speculative if its level of net worth does not cover the desired investment. Therefore the probability ζ for a firm to move from state 2 to state 1 is equal to

$$\begin{aligned} \zeta(t) &= Pr\left[P(t)\ I_2(t) \geq F_2(t)\right] = \\ &= Pr\left\{a\tfrac{[r+f_2(n^c(t))]P(t)}{i(t)} \geq F_2(t)\right\}. \end{aligned} \tag{17}$$

activities NM:

$$M = L_1(Y) + L_2(r, P_k) + L_3(F) - L_4(NM).$$

With regard to speculative firms, they can move to state 2 if they are able to generate a level of profit sufficient to repay their debt; so that the relative probability of transition ν is given by

$$\begin{aligned}\nu(t) &= Pr\left[\tau w(t) b X_1(t) \geq D_1(t)(1+i(t))\right] = \\ &= Pr\left\{\tau w(t) b G[K_1(t), L_1(t)] \geq D_1(t)(1+i(t))\right\} = \\ &= Pr\left\{\tau w(t) b G\left[K_1(t-\delta t) + a\frac{(r+f_1(n^c(t)))P(t)}{i(t)}, L_1(t)\right] \geq D_1(t)(1+i(t))\right\}.\end{aligned} \tag{18}$$

Let us denote with η the *a-priori* probability for a firm to be in state 1, taking it as exogenous[12]. The transition rates, the probabilities of observing a transition from one state to another in a unit of time, will be then given by

$$\lambda(t) = (1-\eta)\zeta(t), \tag{19}$$

$$\mu(t) = \eta\nu(t). \tag{20}$$

4.2 The system dynamics

We have already defined the micro-states of the process, that correspond to states 1 and 2 for the firms. In order to define the macro dynamics we focus on the occupation numbers, that is in the number of firms which are in one of the states at a given time. These occupation numbers identify the macro-states of the process, that, accordingly, are given by all the possible combinations of N_1 and N_2 satisfying the constraint $N_1 + N_2 = N$. In this way, their stochastic dynamics can be conveniently described by a master equation (Kubo et al., 1978; Aoki, 2002). Using N_z to denote the occupation number for the state z, the master equation can be expressed as

$$\frac{dPr(N_z,t)}{dt} = \lambda Pr(N_z-1,t) + \mu Pr(N_z+1,t) - (\lambda+\mu)Pr(N_z,t) \tag{21}$$

where $Pr(N_z,t)$ indicates the probability of observing an occupation number equal to N_z in state z at time t. This ordinary differential difference equation for $Pr(N_z,t)$ allows us to describe the stochastic dynamics of the occupation numbers by identifying the components of the stochastic process that governs their evolution. To this end Aoki (2002) suggests splitting the state variable N_z into its drift (m) and diffusion (s) components, according to

$$N_z(t) = Nm + \sqrt{N}s. \tag{22}$$

At this stage it is possible to apply the method detailed in Di Guilmi (2008) and Landini and Uberti (2008) to obtain the dynamics for m and s. First, by means of lead and lag operators, probability fluxes in and out of the states can be treated as homogeneous. Then, the Taylor series expansion of the modified master equation identifies a Fokker-Planck equation for the transition density

[12] As demonstrated in Aoki (2002) the solution of the master equation provides also a possible endogenous formulation for the probability η. We do not apply this result here as it is not essential in the present study.

of the spread $Q(s, \tau)$ depending on the trend and the diffusion of the process according to

$$\frac{\partial Q}{\partial \tau} - N^{1/2}\frac{dm}{d\tau}\frac{\partial Q}{\partial s} \approx \left[-N^{1/2}\frac{\partial}{\partial s}\alpha_1(m) + \frac{1}{2}\left(\frac{\partial}{\partial s}\right)^2 \alpha_2(m)\right] Q(s, \tau), \qquad (23)$$

where:

- $Q(s, \tau)$ is the transition density function of the spread s denoted with respect to τ, which represents the time rescaled by the factor N, so that $\tau = tN$;
- α_n is the n^{th}-moment of the stochastic process for s;
- $m = \frac{N_z}{N}$ is the state variable, indicating the proportion of firms of type z in the total population of firms.

The asymptotic solution of (23) leads to the system of coupled equations[13]

$$\frac{dm}{d\tau} = \lambda m - (\lambda + \mu)m^2, \qquad (24)$$

$$\frac{\partial Q}{\partial \tau} = [2(\lambda + \mu)m - \lambda]\frac{\partial}{\partial s}(sQ(s)) + \frac{\left[\lambda m(1 - m) + \mu m^2\right]}{2}\left(\frac{\partial}{\partial s}\right)^2 Q(s), \qquad (25)$$

where the first is an ordinary differential equation the solution of which is the drift of the process N_z, while the partial differential equation (25) describes the evolution of the density of the random spread s around the drift. As one can see from (24), m converges to the steady state value m^* given by

$$m^* = \frac{\lambda}{\lambda + \mu}. \qquad (26)$$

Then, directly integrating equation (24) we find that

$$m(\tau) = \frac{\lambda}{(\lambda + \mu) - \omega e^{-\vartheta\tau}} \qquad (27)$$

where

$$\begin{cases} \omega = 1 - \frac{m^*}{m(0)}, \\ \vartheta = \frac{(\lambda+\mu)^2}{\lambda}. \end{cases} \qquad (28)$$

Equation (27) describes the evolution of the fraction m of firms and we see that it is fully dependent on transition rates. The solution of the equation for the density of the spread component yields the limit distribution function

[13] The calculation is fully detailed in Di Guilmi (2008).

$\bar{Q}(s) = \lim_{\tau\to\infty} Q(s,\tau)$ for the spread s, determining, in this way, the long run probability distribution of fluctuations, namely

$$\bar{Q}(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \tag{29}$$

where $\sigma^2 = \frac{\lambda\mu}{(\lambda+\mu)^2}$. Equation (29) is a Gaussian density whose parameters are dependent on the transition rates.

4.3 Analytical description of the model

At this point we are able to identify the two dynamical variables that drive the dynamics of the economy: the first is capital accumulation that reflects investors' expectations and animal spirits, and the second is the underlying stochastic dynamics of the proportion of speculative firms. These two dynamical variables are connected since the transition rate λ is a function of the level of investment I_2 and the aggregate investment depends on the shares of the two types of firms. Taking as state variable the share of speculative firms $n_1 = \frac{N_1}{N}$ whose average trend is given by equation (24), we can write

$$\begin{cases} dn_1(t) &= \left\{\lambda n_1(t) - (\lambda+\mu)[n_1(t)]^2\right\} dt + \sigma\, dW \\ dK(t) &= I(t)dt - vK(t-\delta t) = \\ &= N\left\{[aP_{k1}(t)]n_1(t) + [aP_{k2}(t)][1-n_1(t)]\right\} dt - vK(t-\delta t) \end{cases} \tag{30}$$

where dW is a stationary Wiener increment and $\sigma\, dW$ is the stochastic fluctuation component in the proportion of speculative firms, coming from the distribution (29). These dynamics can then also identify the evolution of employment and aggregate output.

In order to put major emphasis on the role of the proportions of the two types of investors and firms we can substitute equations (6) and (13) into the second equation in (30). Assuming that $\mathbb{E}[\tilde{\varpi}] = 1$, this equation may be expressed as

$$I(t) = \frac{N\ a\ P(t)}{i(t)}\left[r + n_2(t) + n_1(t)\left(2n^c(t) - 1\right)\right]. \tag{31}$$

Equation (31) sheds light on the dynamics of the agent based model and, in particular, it analytically represents the fact that the effect of the proportion of the number of speculative firms depends on market expectations. As long as the chartists are the majority, a rise in the proportion of speculative units causes a positive variation in investment due to the expected rise in the asset price combined with the high market valuation of their equities. When expectations change and the fundamentalist investors prevail, the proportion of speculative firms has a negative effect on investment.

A flow chart of the model is displayed in figure 4.3. As the chart shows, the key variable in the entire story is ρ. The left part of chart summarizes the stages through which it influences the equilibrium prices in the financial market. The right side details the determination of firm profits and thus their capacity to finance future investment with internal funds. Both sides influence the dynamics of aggregate capital.

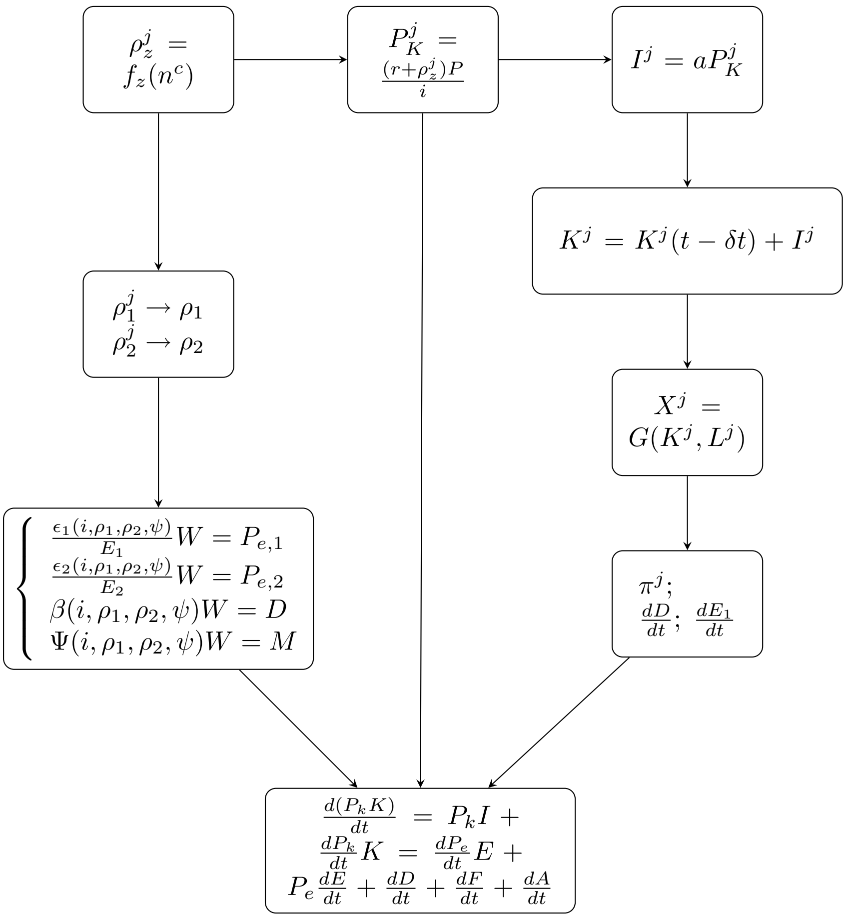

Figure 2: Flowchart of the model. Time indices are omitted.

5 Simulations

5.1 Specification of functional forms

Given that the supply of labour is infinitely elastic and the output/labour ratio b is constant, it is possible to define the production function just as a function of capital:

$$X^j(t) = \varphi \, K^j(t) \tag{32}$$

where the output/capital ratio φ is a constant parameter.

The random variables n^c and $\tilde{\varpi}$ are assumed to have a uniform distribution. As a consequence of these hypotheses the transition probabilities can be specified in term of the known probability function of n^c as

$$\zeta(t) = F(n^c_\zeta) = Pr\left\{ n^c(t) \leq r\,\varpi - \frac{F_2(t)\; i(t)\; \varpi}{P(t)\; a} + 1 \right\} \tag{33}$$

$$\nu(t) = 1 - F(n^c_\nu) = Pr\left\{ n^c(t) > \varpi \left[\frac{i(t)}{P(t)\; a} \left(\frac{D_1(t)(1+i)}{\tau w(t) b \varphi} - K_1(t-\delta t) \right) - r \right] \right\} \tag{34}$$

where $\varpi = \mathbb{E}[\tilde{\varpi}]$. Equations (33) and (34) come from (17) and (18) with ρ_z substituted by (13). The expressions on the right hand sides are the critical levels in the proportion of chartists n^c_ζ and n^c_ν required for a firm to shift from one group to the other.

The functions ϵ_z and β of system (16) are formulated as logistic functions, in order to ensure meaningful values of the proportions of wealth invested in the different activities. The shares of wealth invested in hedge firm equities, speculative firm equities, debt and money are assumed to be positively related to, respectively, ρ_2, ρ_1, the rate of interest i and the parameter ψ. The shares are thus given by

$$\epsilon_1(t) = \frac{1}{1 + e^{i(t)+\rho_2(t)+\psi-\rho_1(t)}}, \tag{35}$$

$$\epsilon_2(t) = \frac{1}{1 + e^{i(t)+\rho_1(t)+\psi-\rho_2(t)}}, \tag{36}$$

$$\beta(t) = \frac{1}{1 + e^{\rho_1(t)+\rho_2(t)+\psi-i(t)}}, \tag{37}$$

$$\Psi(t) = \frac{1}{1 + e^{i(t)+\rho_1(t)+\rho_2(t)-\psi}}. \tag{38}$$

The parameter ψ is kept fixed. Therefore the system (16) becomes:

$$\begin{cases} P_{e1}(t)E_1(t) = \dfrac{W(t)}{1 + e^{i(t)+\rho_2(t)+\psi-\rho_1(t)}}, \\ P_{e2}(t)E_2(t) = \dfrac{W(t)}{1 + e^{i(t)+\rho_1(t)+\psi-\rho_2(t)}}, \\ D(t) = \dfrac{W(t)}{1 + e^{\rho_1(t)+\rho_2(t)+\psi-i(t)}}, \\ M(t) = \dfrac{W(t)}{1 + e^{i(t)+\rho_1(t)+\rho_2(t)-\psi}}, \\ W(t) = P_{e1}(t)E_1(t) + P_{e2}(t)E_2(t) + D(t) + M(t). \end{cases} \tag{39}$$

The mechanism for entry of new firms is stochastic. In every period a random number drawn from a uniform distribution with support $[0, 1]$ is assigned to each potential new firm; if this number is bigger than the normalized variation of aggregate output observed in the previous period the firm becomes active. The variation is normalized such that a variation of $+12\%$ is equal to 0 and of -12% is equal to 1. The typical configuration of parameters is: $a = 4$; $\phi = 1$; $c = 7$; $\psi = 0.25$; $v = 0.009$; while, as far as the supports for the random variables are concerned we choose $n^c \in [0, 1]$; $\tilde{\varpi} \in [0.01, 1.99]$. A control is introduced in order to ensure that $\phi i \leq 1$. These parameters have been calibrated in order to obtain the best performance in terms of replication of empirical data. The values of ρ_z are the mean of the ρ^js included between the 10^{th} and the 90^{th} percentiles within each cluster of firms. The starting values are chosen so that all firms have an initial endowment of internal funds and the initial interest rate is set to 0.1.

5.2 Simulation results

Simulations are performed by implementing two separate procedures, one agent based and the other for the stochastic dynamics, and each produces its own dynamics of proportions of firms and capital accumulation. The two procedures are linked as the exogenous shock of the variation in n^c is the same for both and the mean-field variables ρ_z, obtained from the ρ^j of the agent based simulation, are the inputs for the stochastic approximation. Then, the routine for the agent based model is replicated with the two representative firms for each state, obtaining dynamics driven by the stochastic system (30).

The transition probabilities are normalized by taking the theoretical maximum and minimum values of the right hand side of the inequalities in (33) and (34). We performed simulations for different numbers of periods and the results have been verified by running $1,000$ Monte Carlo replications for each simulation. A period can be considered as a year and the average interest rate for each replication is about 11%.

The model replicates well some quantitative features of US economy as far as the relationships of GDP dynamics with market capitalization and business debt are concerned. The results are displayed in table 2.

The model is also able to replicate some statistical regularities that are observed in real data. The distribution of variations in the aggregate output is well approximated by a Weibull distribution (Di Guilmi et al., 2005) and the distribution of firms' growth rates is well fitted by an exponential PDF (Stanley et al., 1996). The size of speculative firms is distributed according to a Pareto law while the distribution of hedge firms, on average larger and more dispersed, is well fitted by a lognormal one. The overall distribution of size appears therefore as a bimodal distribution.

Figure 3 reports the dynamics of the capital and the share of speculative firms. For both variables the stochastic approximation satisfactorily mimics the results produced by the agent based simulation. The dynamics of capital (and consequently of aggregate production) displays a long term upward trend.

Variable	Empirical evidence	Simulations
Correlation market cap.-GDP (1929-2000)	0.7000	0.6962
Mean market cap.-GDP ratio (1929-2000)	0.6294 ± 0.3648	0.6141 ± 0.2550
Correlation debt-GDP (1950-2007)	0.8612	0.8051
Mean debt-GDP ratio (1950-2007)	0.5181 ± 0.1423	0.5208 ± 0.1658
Variance in GDP fluctuations (percentage, 1950-2007)	0.0022	0.0022

Table 2: Results from Monte Carlo simulation of the model and comparable evidence for the US. Sources: market capitalisation 1929-2008: CRSP data set; GDP 1880-2007: International Monetary Fund; business debt: Federal Reserve Bank of St. Louis. GDP fluctuations are calculated as variations on the long term GDP trend.

Within this trend, long cycles of a duration varying between about 10 and 30 periods and smaller variations (from one period to another) are identifiable. The length and the amplitude of the cycles are determined by the underlying debt cycle (see figure 4). During periods of accelerated growth, the proportion of speculative firms and aggregate debt rise. Consistently with Minsky's model, growth and the accumulation of debt increase until the most indebted speculative firms begin to fail, reducing the amount of capital and the aggregate wealth in the system. The amount of available credit reduces, causing the demise of other speculative units. This downward spiral ceases when all the firms in the relatively worst financial condition have collapsed, allowing the cycle to start again. The dynamics of bankruptcies and capital are compared in figure 5.

Despite the fact that the variance of annual fluctuations of aggregate product is the same, figure 3 displays a more irregular pattern for our simulated economy than the one typically observed in real data, with long period of negative growth. Given the quantitative similarities for correlations of the time series, aggregate production dynamics and growth rates of firm distribution between empirical and simulated data, these long periods of depression that the model displays can be interpreted as the missing contribution of sectors of the economy which are not taken into consideration in the model, in particular the household and public sectors[14]. Historically this contribution consisted in the shift of debt from the productive and financial sectors first to the household and then to the public sector, according to a pattern that has become evident during the recent global financial crisis. In recent decades in the US, and to a lesser extent in all other developed economies, the growth of demand has been sustained by a remarkable increase of household debt. Due to the restrictions in the credit market in the wake of the crisis and to the ensuing recession, firms and households started a

[14]Note also that here the level of productivity is assumed to be constant.

process of deleveraging which forced national governments to take on part of the private sector's liabilities or to directly sustain demand, shifting the private debt to the public sector. This aspect is further analysed in figure 6 which displays the debt series generated by the model, highlighting the periods of negative variation in production, and how this contrasts with comparable data for the US. In both cases the beginning of a deleveraging process marks a recession. This finding is consistent with the evidence of the countercyclicality of corporate debt presented in Jermann and Quadrini (2009). But while in the US the recessions are typically brief and always shorter than the deleveraging phase, in the model they last for approximately all the period of negative variation in business debt. In the real world household and public sector provided so far a safety net, while in the model deleveraging and depression last until all the weakest productive units exit from the market.

The effect of the investors strategies on liquidity in the economy is well represented by figure 7 which shows the relationship between the proportion of chartists and the level of the interest rate. The correlation appears to be negative for a small proportion of chartists and positive for large values of n^c. This may be due to a liquidity effect in the right part of the graph (investors own an increasing quantity of bonds and then there is a positive liquidity effect). This effect is more than balanced by the increasing demand for credit as the proportion of chartists (and consequently the level of investment for speculative units) grows.

The possible areas of intervention for the policy maker to reduce the volatility of the system are identifiable in the limitations to the creation of liquid assets and in the regulation of bankruptcy. These two areas are represented respectively by the parameters ψ, which quantifies the capacity of the system to create endogenous money, and c, which is the maximum debt ratio allowed to avoid failure.

As shown by figure 8, the degree of financial innovation and the consequent capacity of the system to create endogenous credit plays a key role in generating the booms. The availability of easily tradable financial instruments pumps liquidity into the system augmenting the level of wealth $W(t)$. Credit becomes cheaper and the accumulation of capital occurs at a faster rate. A bigger stock of liquid assets in the market makes possible the accumulation of a larger stock of capital in the long run. The downside of this is the much larger volatility and the associated long periods of contraction. From this perspective a regulatory framework for securitization and in general for financial derivatives may reduce ψ and be effective in reducing the volatility. Interestingly, for high values of ψ (above 0.6), the system may become unstable and subjected to waves of failures that involve almost all the active firms.

A larger value of the parameter c corresponds to an easing in the position of heavily indebted firms[15] or to a temporary support to those in a critical condition. The model demonstrates that larger debt ratios can lengthen the

[15]Chapter 11 in the USA and other similar legislations in other countries may be regarded as an example.

positive trend but, as a consequence, the period of distress is longer, increasing the uncertainty in the system (figure 9). These results are consistent with the findings of Suarez and Sussman (2007) that a softening in the bankruptcy law ensures a faster growth at the expense of long term stability.

The degree of uncertainty in the present model can be quantified by the parameter a, which measures the reactions of firms to market expectations, and by the possible changes in investor strategies, captured by the distribution of n^c. As a grows the cycles become more regular, longer and of larger amplitude. Beyond a certain threshold ($a > 10$) the positive long run trend virtually disappears and only long fluctuations are observable. A reduction in the support for the distribution of n^c or a less dispersed distribution (truncated normal rather than a uniform distribution) reduces the average interest rate, allowing a more sustained growth with a slower accumulation of debt.

As regards the choice of firms between equity and debt as source of financing, figure 10 shows that, as the proportion of investment financed with equity (measured by ϕ) increases, the system becomes more stable. The accumulation of capital improves despite the fact that, for low shares of debt financing, private wealth shrinks.

6 Concluding remarks

In this paper we study the transmission of shocks from the financial sector to the real economy, along the lines of Minsky (1975) and Taylor and O'Connell (1985). We provide a consistent microfoudation for Minsky's theory using two different methods, one numerical and the other analytical, to deal with the issue of heterogeneity. This feature has so far prevented a wider diffusion of Minsky's ideas, since, until recently, this class of models could be discussed only in macroeconomic terms. The present model involves firms that are heterogeneous with respect to size and financial conditions and can generate two types of dynamics. One is agent based and allows a numerical solution which replicates some stylized facts of a real economy; the other is a stochastic approximation that can be solved analytically. This latter turns out to be capable of satisfactorily mimicking the outcomes of the agent based model with a much higher degree of heterogeneity.

The simulation results show dynamics consistent with Minsky's intuition: there are boom periods, in which the economy grows at a rate significantly higher than its long run trend. At the same time the availability of credit leads firms to take on more debt. When the units in the worst condition begin to fail the process reverses, causing a depression. This pattern is reproduced cyclically, revealing a structural fragility of the system. The economy can be stabilised by reducing its capacity to create endogenous money and the maximum debt ratio allowed.

Such an approach is not possible within the traditional economic paradigm. Indeed, even though in principle allowing a bottom-up approach, it is by construction unsuitable to deal with this problem for two main reasons. The first is the representative agent hypothesis, which is hardly compatible with the Financial Instability Hypothesis (that is formulated considering different types of firms with respect to their financial structure) and cannot involve phenomena like insolvency and bankruptcy. The second reason is that, according to the neoclassical view, financial markets are not even potentially a factor of instability as, on the contrary, they are supposed to stabilize the economy, absorbing temporary disequilibria in real markets by means of derivatives and futures.

The framework presented here appears then to be an efficient tool to analyse the effects of the instability in financial markets on the real sector of the economy and the related issue of the roles of the private and public sectors in avoiding depressions.

Further development of this approach, in particular a more refined modelling of the generation of investors' expectations, may be useful for the identification of the conditions under which the system generates speculative bubbles and how they burst. The model may also be extended to include various forms of speculative behaviour and the banking sector in the intermediation of credit. Another possible application regards institutional aspects such as government policies, fiscal and monetary, and the study of the possible effects of a regulatory framework.

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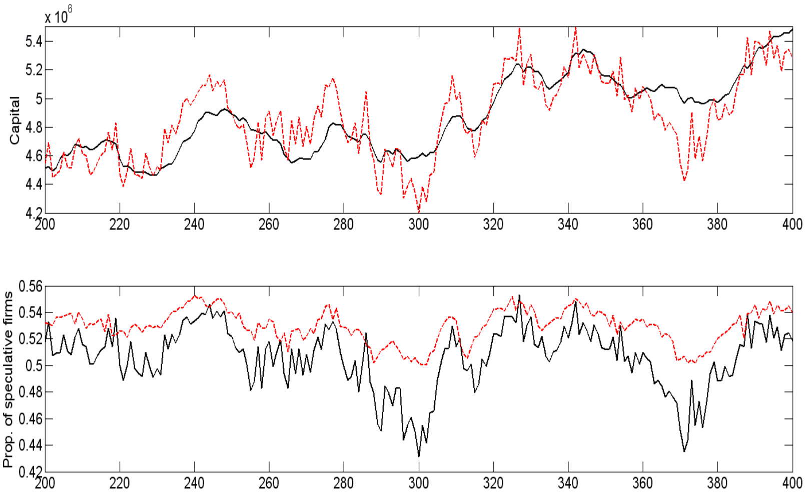

Figure 3: Different dynamics of capital (upper panel) and share of speculative firms (lower panel). Simulation of the agent based model (continuous line) and the endogenous stochastic dynamics (dashed line).

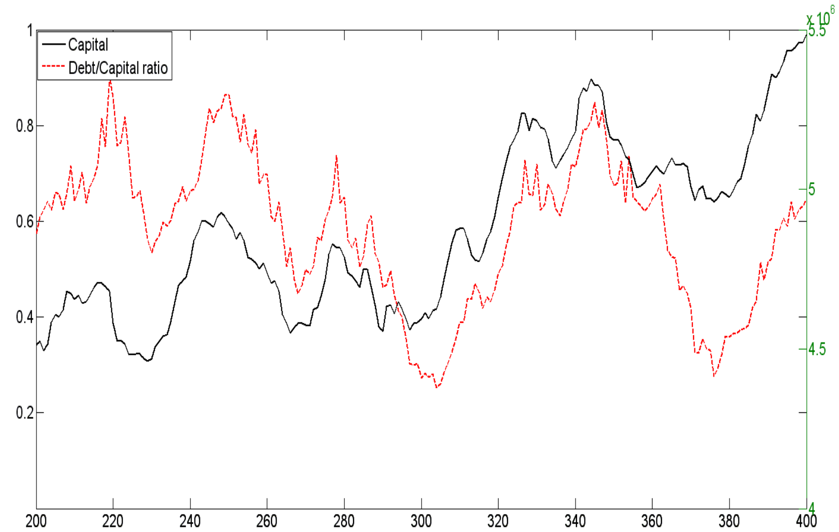

Figure 4: Debt/capital ratio (left axes) and aggregate capital (right axis). Simulation of the agent based model.

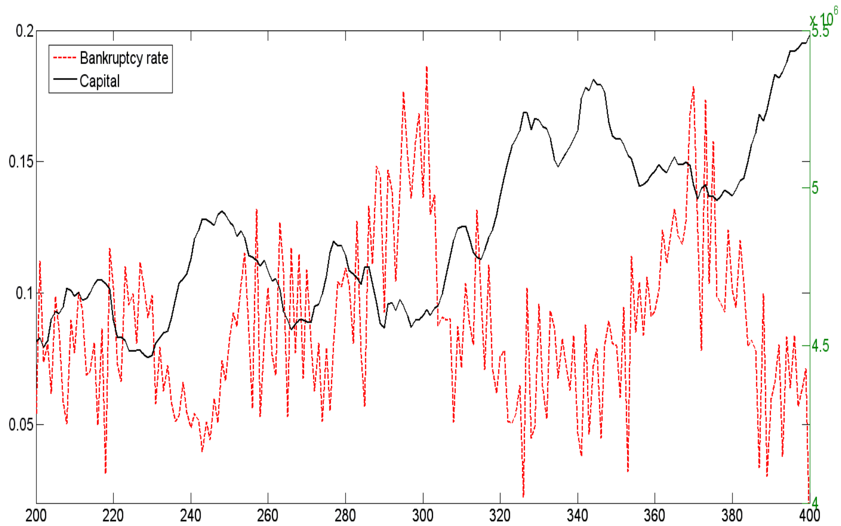

Figure 5: Rate of bankruptcy (left axes) and aggregate capital (right axis). Simulation of the agent based model.

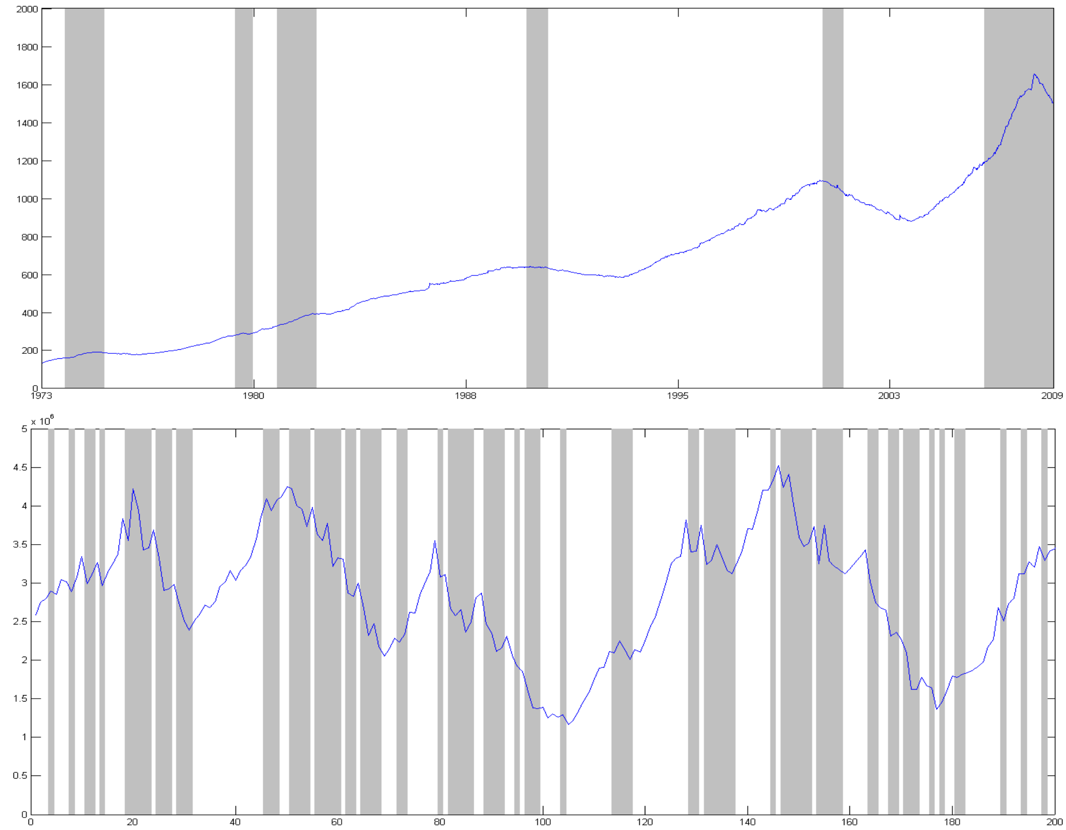

Figure 6: Upper panel: Business debt dynamics for the US (billions of dollars) and recessions (grey areas). Lower panel: comparable results from simulations of the the agent based model.

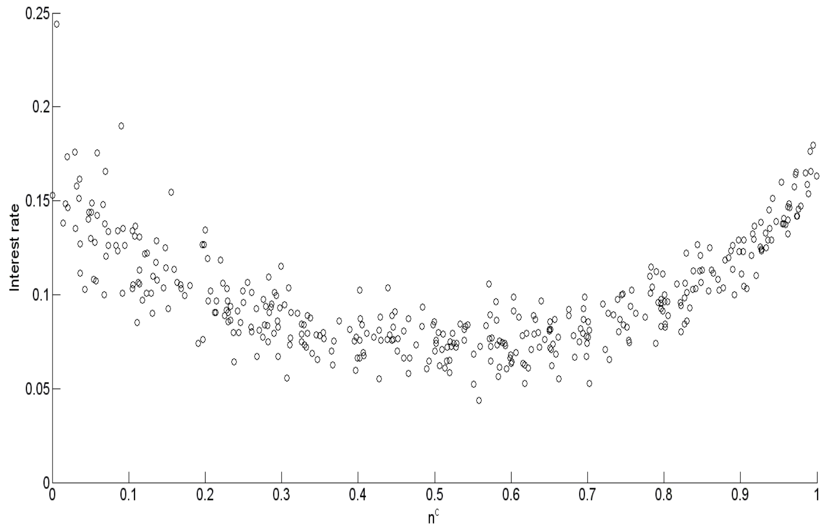

Figure 7: Scatter plot of proportion of chartist n^c and the interest rate. Simulation of the agent based model.

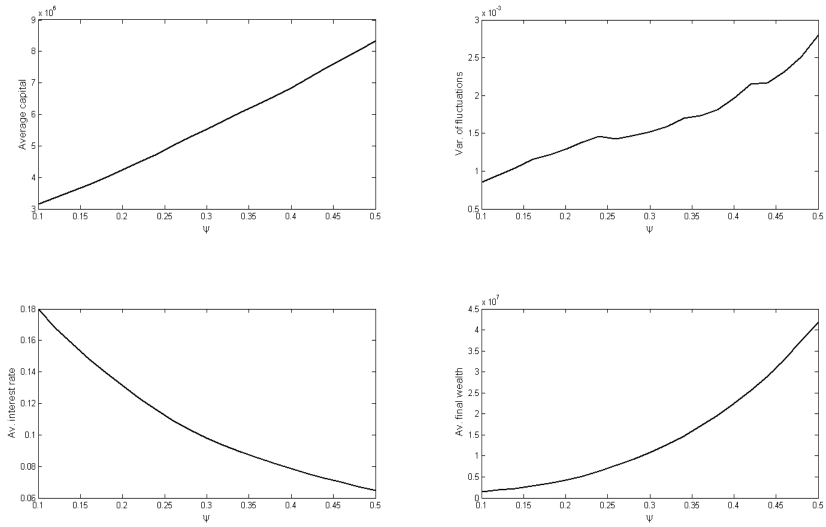

Figure 8: Average aggregate capital, variance of fluctuations, interest rate and final wealth for different values of ψ. Monte Carlo simulation of the agent based model.

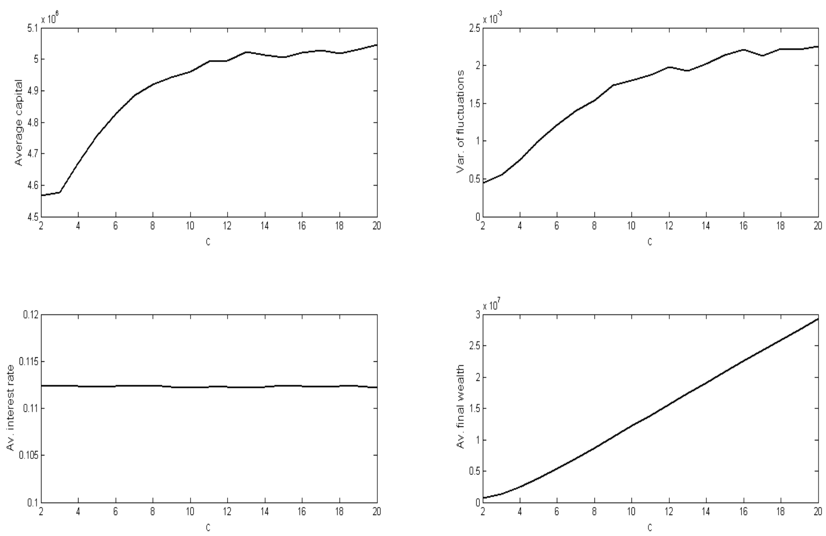

Figure 9: Average aggregate capital, variance of fluctuations, interest rate and final wealth for different values of c. Monte Carlo simulation of the agent based model.

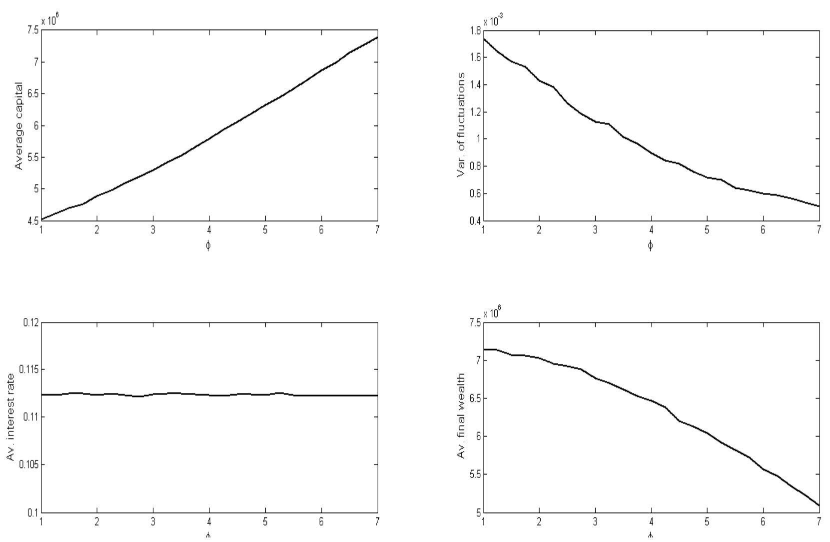

Figure 10: Average aggregate capital, variance of fluctuations, interest rate and final wealth for different values of ϕ. Monte Carlo simulation of the agent based model.