A Brief History of Equality

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Abstract: Trends in distribution of wealth and income have profound implications for economic and political stability in societies. We point out an "Iron Law" of distribution: Future inequality of wealth is decreasing in the income elasticity of family size. It is an "Iron Law" not because it is always the most significant influence on the income/wealth distribution at any point in time—it may not be—but because its effects are relentless, systematic and cumulative. Since transfers from parents to children are include a substantial 'private good' component, the transfers received by any child is a negative function of the number of siblings other things being equal. Accordingly, if more wealthy families have more children than less wealthy, then children in more wealthy families will receive a smaller share of the family wealth than children in less wealthy families; and this process over time will operate as a brake on concentration of wealth. But if more wealthy families have fewer children, the process will lead to rapidly concentrating wealth. As we show, this consideration, in surprisingly accurate and modern form, was suggested by Smith (1776) in anticipating Malthus (1798), thus pointing up a connection between current concerns with income inequality and classical discussions of population growth.

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I Introduction

I.A. Overview

Distribution and inequality of wealth and income are perennial concerns, both for academic economists and for practical politics. But interest in the sources and consequences of inequality has recently burgeoned, perhaps because of (apparent) trends towards increasing inequality within many countries[1] - including Australia (Atkinson, 2015; Atkinson and Leigh, 2007; Burkhaur, Hahn and Wilkins, 2015; Cobb-Clark and Hildebrand, 2009; Cobb-Clark, 2010; Cowan, 2014; Leigh, 2007; Leigh, 2013; Mendolia and Siminski, 2016; Wilkins, 2014 and Wilkins, 2015).

Such inequality will, according to Picketty (2014), pass on to future generations. For example, Picketty predicts that in France, up to 90% of all personal wealth will soon be inherited – returning the wealth distribution to something like what is was in the 19th century!

The current paper is directed at the inter-temporal path of inequality (both of income and wealth); and in particular at one of the causal factors that seems likely to be at work over the long run.

Specifically, we aim to expose and analyse one explanation for long-term trends in inequality. The mechanism we have in mind is so fundamental, and so obvious that it seems (paradoxically) nearly to have escaped attention. We refer to the mechanism as the Iron Law of intergenerational transmission of dispersion (Iron Law for short).[2] The Iron Law is simply

[1] For a detailed examination of the phenomenon itself see Atkinson et al (2011); for a general introduction, see Blinder (1973).

[2] The first use of "iron law" in this context appears to have been from Goethe, in 1783, and then used by Malthus (1798). Karl Marx in Part II of his "Critique of the Gotha Program," (1875), referring to Lasalle's "Iron Law of wages," which Marx claimed was borrowed from Goethe's line from "DAS GÖTTLICHE" : "As great, eternal iron laws dictate, we must all complete the cycles of our existence." Lasalle was reformulating the Malthusian (Malthus, 1802) that the working population would be tightly constrained by the food supply, and that wages would adjust to ensure that the constraint was binding. Roberto Michels (1911) later advances what he calls "The Iron Law of oligarchy." We use the phrase not to be grandiose, but to emphasize that the "law" has mechanistic features and tightly constrains events.

this: *Future inequality of wealth is decreasing in the income elasticity of family size.* We call it an "Iron Law" not because it is always the most significant influence on the income/wealth distribution at any point in time—it may not be—but because its effects are relentless, systematic and cumulative. The overall tendency for inequality to increase, or decrease, over successive generations operates consistently according to the sign of that elasticity, and the speed of the effect is determined by the magnitude of that elasticity, whatever else is going in the economy. Like other Iron Laws in the social sciences, this one operates 'invisibly' in Adam Smith's sense – the aggregate patterns it produces are in no way the result of the intentions of the participating agents.

The theory is explanatorily 'parsimonious' in the sense that its operation depends on nothing more than elementary logic and one (robust) empirical assumption about the human species. That assumption is simply that parents and testators care more about their own children than they do about others' children. The consequence is that parents make transfers to their own children during the parents' lives, and then leave their estates primarily to their own children.[3]

The Iron Law is derived, then, from parents' transfer to children of ordinary private goods, which by definition are 'rival in consumption'. This implies an economic analogue of 'sibling rivalry', which we shall term 'sibling rivalness' to distinguish it from its familiar psychological cousin. Sibling rivalry is contingent: it may be common but it is not logically necessary. Sibling rivalness is a logical necessity, because the magnitude of the transfers received by each offspring is *ceteris paribus* a negative function of family size.

[3] In the absence of such transfers in the early years of life, it is doubtful whether the human race could survive. In this respect, the human species is like others in which offspring are not self-sufficient at or close to birth. In the human case, of course, what counts as "self-sufficiency" is itself historically dependent. And it is clear that the psychological factors of natural affection (and norms of parental duty) that cause parents to look after their children do not disappear once children are self-sufficient: transfers proceed up to and including parental death. For evolutionary purposes of course what matters is survival rate to age of reproduction. An interesting model of the impacts of different motivations for intergenerational transfers is Zilcha (2003).

It is easy to conflate the fact of inheritance—the passing of a fortune to one's children—with the particular mechanism we identify, which derives from inheritance interacting with family size as a function of wealth. To exemplify, suppose that the income elasticity of family size in a particular time period is positive – the richer the family, the more surviving offspring, *ceteris paribus*. Then the intra-family lifetime transfers (the sum of nutrition, education, inheritance, etc.) to offspring will be divided among a larger number of claimants in richer families. This implies that the ratio of the incomes of rich families compared to poor families in a period will be greater than the ratio of the incomes of their offspring, because the wealthy testators will have their wealth divided among more heirs. This effect will substantially mitigate, and might even reverse, tendencies in the economy that might otherwise concentrate wealth in fewer hands. So long as wealthy transferers have more recipients, the operation of the Iron Law is anodyne: Dispersion in the distribution of income over time is inherently self-limiting.

The problem is that the Iron Law also operates if income elasticity of family size has a negative sign. But then the effect is far from anodyne. In fact, if increasing income is associated with smaller family size, some of our basic assumptions about the structure of democratic societies may prove untenable. Concentrated wealth becomes more concentrated, and the variance of income increases rapidly. This effect is not bounded-, but is relentlessly accelerated by the interaction of family size, intra-family transfer, and subsequent family size.

This logic suggests that the empirical sign of the income elasticity is of central importance. And the trend in different societies shows a striking—and for the reasons described above, in some cases sinister—pattern. Society after society has reached, at different times, but at approximately the same stage of development, a point where income increases come to be associated with fewer, rather than more, children. For example, Becker (1981) and Becker and Tomes (1994) have suggested that, in modern times, the higher opportunity cost of children for families with greater human capital will mean that richer families will tend to be smaller. Österberg (2000) and Lindahl (2008) offer some empirical estimates of the effects of this changing elasticity on "income mobility," and support the idea that the pattern is important, though worrisome.[4]

Our contribution is to nail down the underlying logic of the formal relationship rather than relying on empirical estimates. Further, it turns out that there is precedent for our theoretical claims: Smith (1776) foreshadows Malthus (1798) in emphasizing the positive causal connection between income and population – which in Smith is driven by the rate at which "a great part of the children which … fruitful marriages produce" are "destroyed" (op. cit. p.98). Now it is a familiar observation at the level of national comparisons that the rate of growth in income during the Malthusian era was associated with increasing population; but in the modern era, higher income is associated with lower population growth. Supposing that the national experience is mirrored at the individual level, the obvious conjecture is that in the Malthusian era richer families had larger numbers of surviving children, while poorer families had fewer surviving children, because of the differential capacity to protect offspring against malnutrition and disease.

The importance of the dynamic process we adumbrate is hard to overstate. At its base is a ,key observation: in the modern era, as survival rates rose, family size became a 'demand-side' phenomenon (dominated by the birth rate) rather than a supply-side phenomenon (dominated by the survival rate). In this sense, Becker's emphasis on demand-side

[4] Our claim that this effect, in its full force, has been overlooked is borne out by Atkinson (2015, pg. 159) who cites us (Brennan et al., 2014) when referring to the mechanism, though not the name, of the Iron Law. The idea is also mentioned briefly in the context of inherited wealth in a survey chapter, Davies and Shorrocks (2000, pg. 622), "… if the wealthy consistently have fewer children, inherited wealth can become continuously more unequal." However, we would argue that the Iron Law has a far wider jurisdiction than mere inheritance, operating as it does via in vivo transfers or via beneficial parental attention, which differentially enhances the human capital of children in small families.

considerations (in determining the choice between quantity and quality of offspring) becomes increasingly relevant.

On this basis, the Iron Law suggests a simple reduced-form "history of equality". In the Malthusian era, the Iron Law tended to produce ever greater equality: it suppressed the incomes of the offspring of richer, larger families vis-a-vis the offspring of poorer, smaller ones. In that sense, the Iron Law was favourable to the emergence of an ever-expanding 'middle class." In the modern era, by contrast, as wide-spread availability of contraceptive measures meant the predominance of demand-side considerations in the determination of family size, the iron law's effects are reversed: the forces of differential family size across income classes involve ever greater inequality over time (whether measured in terms of wealth or income).

This description of the iron law's operation involves an independent role for income as a *determinant* of family size – in part because the economic literature[5] suggests independent reasons why income should play an independent causal role. However, it is worth noting that the iron law could in principle operate without income exercising any causal influence on family size.

To illustrate, suppose in a situation of identical incomes in period, there is some random variation in family size. Sibling rivalness effects will still be in operation and ceteris paribus children from smaller families will receive larger intra-family transfers than children from larger families. In the next generation, the same random effects will be in evidence: the

[5] Both Smith (1776) and Malthus (1798) offered speculative reasons why this might be true. The modern economic and demographic work on this question suggests two vectors of influence: capacity and taste. Capacity is related to the ability of a family to provide food and medical care, meaning that more offspring will survive, or that mothers may survive childbirth more often. Taste is the decision to have children, so that if economic factors matter at the margin the effects of price, income, and other variables will be in the expected directions. Becker's method is important, because he ruled out routine resort to "changes in preferences" as an explanation for observed changes. That is not to say that tastes, culture, and societal norms are invariant. But given Becker's general methodological stance, explanations based on changes in observable parameters should be privileged.,.

proportion of rich individuals with a 'history of small families' will tend to increase over time, whatever other factors in income determination are in play.

I.B. Caveats

Some clarifications and caveats should be inserted at the outset.

1. The mechanism we focus on makes an implicit assumption of perfect assortative mating. Mating patterns are clearly important and in the limit, systematic offsetting mating could undermine the entire mechanism, though we find this an implausible scenario. Indeed, assortative mating is becoming more common alongside increased female participation in higher education during the 'matching' age (Economist, 2012).

2. The Iron Law involves no claim of any genetic connection between the income-earning capacities of parents and children. Indeed, any such would presumably operate as an intra-family 'public good' to all siblings; and hence not be subject to the sibling rivalness effect. In this sense, any claim concerning a putative relation between income elasticity of family size and the aggregate quality of the genetic pool is entirely alien to the spirit of this paper.

3. Although we have illustrated the central property of sibling rivalness via appeal to 'economic goods' narrowly construed, we do not think the phenomenon is so restricted. For example, we take the apparently robust empirical finding in the sociological literature that educational performance is negatively correlated with family size (other things equal)[6] to emphasize that parental attention and energy in relation to their children's education is no less 'rival' in the economic sense than food or Princeton fees.

4. This is not an empirical paper. We make no attempt here to assess how significant the iron law effects are in the current situation. Our focus is on the logic of the mechanism itself. We consider that mechanism sufficiently interesting to be worth independent conceptual scrutiny.

[6] See for example Blake (1989), Alexander and Cherlin (1990) and for the developing world, Dang and Rogers (2013).

5. Although bequests are one important mechanism of intergenerational transfer, they are not the only one. Over most of human history, because humans do not reach material independence for an extended period (which may itself be endogenous), much the most significant intra-family transfer receipts for most individuals are those that occur over the first decade or so of the individual's life. We want to emphasise that bequests need not be the main game in the operation of the Iron Law – the apportioning of importance between bequests and *in vivo* transfers awaits future empirical research. .

I.C. Outline

The lay-out of the discussion is as follows.

In section II, we illustrate the effects of the iron law by taking a Taylor series approximation of the relation between child income and parent income – and allowing explicitly for variance in the relation between family size and parental income. We find a parsimonious relationship between inequality, the intergenerational elasticity of earnings, and fertility.

The operation of the iron law would be perhaps be of less interest if the relation between income and family size had been stable throughout history. In section III, therefore, we turn to the historical material, using secondary data to try to locate different periods of the value of the income elasticity of family size; and exploiting Malthusian period data to buttress our claims about the importance of income-mediated factors.

Section IV draws together some brief conclusions.

II An Analytic Sketch of the Iron Law

2.1 The inequality recursion

In this section we fomalize the operation of the Iron Law to see how it might affect inequality, measured in a standard way. The setup is deliberately simple, and we focus on a key parameter – the income elasticity of family size. However, we also allow for other influences on family size, which we model by an i.i.d. error.

To abstract from economic growth, which is not *per se* determinative of relative incomes, the distribution of income is normalized over the support (0, 1). We assume that income Y_{it}, in cohort i, is divided equally among individuals in a family, N_{it}, and therefore that income available for the next generation is proportional to per person income:

$$Y_{i,\,t+1} = f\left(\frac{Y_{i,t}}{N_{i,t}}\right) \qquad f\,' > 0. \tag{1}$$

We implicitly assume perfect assortative mating by men and women on the same income level.[7] We choose a tractable functional form that prevents wealth from diluting over time -- that is, a functional form which generates a constant income in the steady state (when $Y=y$ and $N=n$).

$$Y_{t+1} \quad = \left\{ y\left(\frac{n}{y}\right)^{\Phi} \right\} \left(\frac{Y_t}{N_t}\right)^{\Phi} = y^{1-\Phi} \left(\frac{nY_t}{N_t}\right)^{\Phi} \tag{2}$$

Agents live for two periods. Over the first period the number of children born depends on wealth (since agents only work for one period we make no distinction between wealth and income). At the end of the first period, their parents die and their bequests are given by (2). In the next period, they become parents, and make transfers to the next generation. In (3) the number of children born in a period N_{it} is determined by income and by a random i.i.d. error u. The average family size n in (2) is determined only by average income i.e. $n=\alpha+\beta y$.

$$N_t = \alpha + \beta Y_t + u_t \tag{3}$$

We now make some extra (and standard) assumptions in relation to (3). We suppress the i subscript and assume N and Y vary over families:

$$Cov\ (N_t, Y_t) = \beta V(Y_t) \tag{4}$$

$$V(N_t) = \beta^2 V(Y_t) + \sigma^2, \qquad V(u) = \sigma^2. \tag{5}$$

[7] As Fernández Rogerson (2001) point out, assortative mating alone will have some impact on inequality. Formally, asexual reproduction is the same as perfect assortative mating.

For any linear distribution of income, including *Y~Uniform (0,1)*, the Gini coefficient and the standard deviation are very close.[8] The importance of this result lies in the fact that it is much easier analytically to work with standard deviations than with Gini coefficients, and we shall exploit this now.

We take a Taylor series linearization of income in period *t+1* evaluated at the means *n* and *y* and work out its variance.

$$Y_{t+1} = y^{1-\Phi}\left(\frac{nY_t}{N_t}\right)^{\Phi} \approx y^{1-\Phi}\left[\left(\frac{ny}{n}\right)^{\Phi} + \Phi\left(\frac{ny}{n}\right)^{\Phi-1}\left(\frac{nY_t}{N_t} - \frac{ny}{n}\right)\right]$$

$$so \qquad Y_{t+1} \approx y + \Phi\left(\frac{nY_t}{N_t} - y\right)$$

$$\& \qquad V(Y_{t+1}) = \Phi^2 V\left(\frac{nY_t}{N_t}\right) \tag{6}$$

We take another linearization to obtain the variance.

$$V\left[\frac{nY_t}{N_t}\right] \approx V[Y_t] + \left(\frac{y}{n}\right)^2 V[N_t] - 2\frac{y}{n}Cov(Y_t, N_t) \tag{7}$$

We can now substitute the above expressions for $Cov(N_t, Y_t)$ and $V(N_t)$ into (7) and in turn into (6) to obtain the time series process for the variance denoted *V* and its equilibrium value, after a bit of algebra. The resultant equation (8) is the inequality recursion, from which we can derive the steady state in (9). Finally, we recall that the square root of the variance is roughly the same as the Gini, so we also express (9) in that way.

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[8]The Gini coefficient is based on the so called Lorenz curve, which measures cumulative income against cumulative population. A Gini of zero indicates no inequality (see Dorfman, 1979). Let $f_Y=1-(b/2)+bY$ be the linear income pdf over *(0, 1)* with slope *b*, constructed to integrate to unity. The standard deviation of *Y* is $\frac{1}{3}\sqrt{\frac{3}{4}-\frac{b^2}{6}}$ and we use Dorfman (1979) $G = 1 - \frac{1}{E(y)}\int(1-F_y)^2 dy = \frac{1}{3}\left(\frac{1-b^2/20}{1+b/6}\right)$ to obtain the Gini where F_y is the cdf. Now in order to guarantee a positive pdf it must be the case that $-2 \le b \le 2$ so these expressions will not be too far numerically from *1/3* for many admissible values of *b*. The standard deviation and the Gini are exactly $1/\sqrt{12}$ and *1/3* when f_Y is uniform, since *b=0* in that case.

$$V_{t+1} = \varepsilon^2 V_t + \left(\frac{\Phi y \sigma}{n}\right)^2 \quad where\ \varepsilon = \Phi(1-\eta) \tag{8}$$

$$V_{steady\ state} = \frac{\left(\frac{\Phi y \sigma}{n}\right)^2}{1-\varepsilon^2} \Leftrightarrow Gini_{\ steady\ state} \approx \frac{\Phi y \sigma}{n\sqrt{1-\varepsilon^2}} \tag{9}$$

A well as being simple, (9) contains an intuition. We collected the terms $\Phi\ (1-\eta)$ and defined a new pronumeral ε because the latter has an economic meaning. It is the intergenerational elasticity of income defined as the proportional change in income in period t+1 resulting from a proportional change in period t.

$$\begin{aligned}
Y_{t+1} &= y^{1-\Phi}\left(\frac{nY_t}{N_t}\right)^{\Phi} \\
\varepsilon = \frac{\hat{Y}_{t+1}}{\hat{Y}_t} &= \Phi(\hat{Y}_t - \hat{N}_t)/\hat{Y}_t \\
&= \Phi(\hat{Y}_t - \eta\hat{Y}_t)/\hat{Y}_t \\
&= \Phi(1-\eta)
\end{aligned} \tag{10}$$

Thus we have from (9) that through an Iron Law mechanism, steady state inequality is increasing in the intergenerational income elasticity. The estimated value of ε is usually less than unity and typically around 0.4 for developed countries and 0.7 for developing countries (Corak, 2012). Clearly, from (10), as societies have a higher (negative) income elasticity of family size, they will become less mobile (ε will rise).

This positive relationship has been the basis Corak's celebrated 'Great Gatsby curve' (Corak, 2012) and it has been given a sinister interpretation by Krueger (2012) – societies with greater inequality will, as a consequence, suffer less intergenerational mobility in the future. However, the interpretation we offer is not sinister – and we take issue with 'as a consequence'. At least in this model, both a high Gini and a high intergenerational elasticity are a consequence of a third causal factor, namely a high and negative income elasticity of family size.[9]

2.2 Policies Aimed at Inequality

A focus on the income elasticity of family size is clearly most interesting if in fact η is able to change, and so we will presently address the possibility of a sign reversal in history. In the remainder of this section, however, we want to simply take some policy advice from equations (8) and (9).

The first point from (8) is that inequality has a non-zero autocorrelation, so the effects of policies travel into the future for generations. While this is an obvious point, one wonders whether policymakers since the 1980s have given proper weight to the fact that some negative consequences might be generational.

Equation (8) also shows that social mobility is a key factor in the persistence of inequality. Part of this is determined by the deep parameter of the income elasticity of family size, but the parameter Φ is a summary parameter and may be open to policy influence.

From (9) it is clear that steady state mean number of children is negatively related to inequality. The intuition is that N appears on the denominator of nY/N and so as the mean of N drops nY/N is exposed to low draws of N which, in view of this inverse relationship, will boost the children's incomes a lot. To put the point straightforwardly, the sibling rivalness implications of a family having two children versus one child are very much more substantial than the rivalness implications of having five children in a family that already has four.[10] It is

[9] We raise this to be wary of over-dogmatizing the data, but refutation is beyond us because the analytic background is not obviously identical. Corak and Krueger's story is an inter-temporal out-of-equilibrium one whereas the positive relationship in (9) is based on a steady state. Furthermore, by using a simple model with only one period of economic activity per 'parent', we have not made a distinction between wealth and income.

[10] For low numbers of children, a nice line of mathematical reasoning shows that the Iron Law greatly accelerates the removal of inequality in the Mathusian case, to the point where everyone attains the mean in a single generation (with the Gini being zero in this case). We can take the limit of nY/N as α approaches zero in

mere conjecture, but our suspicion that this is rarely a topic of conversation among parents contemplating an extra child is what led us to float the idea, in the introduction, that the Iron Law is invisible in the Smithian sense – that is, invisible to the actors themselves. Even if parents were aware of the impact on existing children of a marginal child birthed to the family, it is far less likely that they would be aware of the effects of their choice on the overall income distribution.

An obvious, though difficult to implement, policy to reduce inequality is to raise the birth rate. Conversely, the Iron Law shows in a way not commonly understood that China's infamous one child policy has contributed to shaking the socialist foundations of that society, by lowering n.

Finally, (9) also shows that the volatility of numbers of children, unrelated to the income effect on numbers of children, will exacerbate inequality. We noted in the introduction that the Iron Law still operates if the income elasticity of numbers of children is zero, and that is borne out by (8) and (9) together. Equation (9) raises the spectre that in some modern economies, the ratio of σ/n may have not changed much, attenuating any negative effects on inequality of a fall in n.

Thus, a policy maker concerned about inequality can be both encouraged and discouraged by the general equation (8). The good news is that because future inequality derives from current inequality, all the standard tools of addressing inequality now through the tax and transfer system will have an intergenerational legacy.[11] Furthermore, the Iron Law operation gives *extra policy teeth* in the Modern Era (where β is negative and so $1-\eta > 1$). That is, a high ε^2 is bad for inequality in the steady state, but ε^2 is also the partial

the Malthusian case (the limit does not exist for the modern era, because of the assumption that n and N must always be positive) and we obtain $nY/N \rightarrow (\beta y)\, Y / (\beta Y) \rightarrow y$.

[11] There is substantial evidence, of the sort produced and reviewed by Roine and Waldenström (2008), and Roine, Vlachos, and Waldenström (2009), that such policies can change both behavior and levels of inequality. Our goal is to identify the underlying tendency in the absence of such institutions.

derivative of (the square of) future inequality with respect to (the square of) current inequality. So away from the steady state any policy measures (outside the model in (8)) that reduce the current V_t will have an even more powerful effect on V_{t+1}. Pulling families in from rich and poor extremes towards the centre of the distribution in the current generation reduces the variation in family size across the income distribution *ceteris paribus* thus attenuating the differential impact of sibling rivalness. However, the existence of σ in the equation is also cause for pessimism. Even if redistributive policy could create a perfectly equal society now, i.e. $V_t=0$, equation (8) shows that the operation of the Iron Law sows the seeds of inequality afresh every generation.

Consideration of the Iron Law puts two standard policies front and centre in any debate about intergenerational inequality. Public education, which tries to take one of the most important forms of a child's capital accumulation out of the reach of the Iron Law, has an obvious appeal. It is not a panacea, though. Parental attention or any other family factors subject to sibling rivalness are complementary inputs into the development of a child's human capital (Blake 1989), even if they are combined in a 'public school production function'. In our model, such a reform could be represented, in a rough and ready way, by a fall in Φ.

Inheritance taxes, too, are worth considering (Davies, 1982) as a means of diluting the pool of resources subject to sibling rivalness. Perhaps inheritance taxes would gain greater appeal if people understood the current implicit system of 'taxation' derived from the Iron Law, where the share lost to siblings depends on the number of siblings present, and the prospective number of siblings.

III Learning from the Past: Changes in the Income Elasticity of Family Size

Our task in this section is to justify the claim that the income elasticity of family size has shifted from being positive prior to the industrial revolution – what we call the 'Malthusian' era - to negative now, in what we call the 'Modern' era.

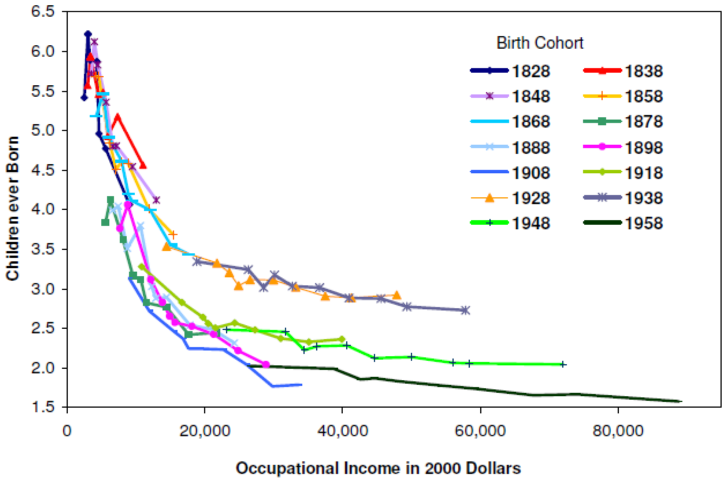
The Modern Era:

Although for many countries, data on the connection between income and fertility is not easy to come by, we illustrate in Table 1, the relation between household wealth (not income) and fertility for a range of countries within the contemporary period (last fifty years say). On this basis, it seems reasonable to conclude that the income elasticity of family size is negative (or perhaps zero, as in the Ukraine) across a range of countries for that period. Reliable data *is* available for the US on the relation between fertility and income within the US back through the nineteenth century [Jones et al (2008), Jones and Tertilt (2008)]: and the relation between income and fertility for various periods is shown in Figure 1. For the US, at least, it seems as if the income elasticity of family size has been negative throughout this period. In other words, in the US, the "modern period", as we characterize it, extends back at least to 1830.

The elasticity for the most recent cohort in the US (the line commencing in 1958) is reported to be -0.22 in Jones and Tertilt (2008). So if we want an equation with income normalized over (0,1) for comparison we can linearize $N = (N_{Y=1})\ Y^{-.22} = (1.6)Y^{-.22}$ around a (normalized) income of unity to get:

$$N = 1.952 - .352.Y \qquad\qquad 0 < Y < 1. \qquad\qquad (11)$$

Figure 1: Fertility by Occupational Income in 2000 Dollars



Source: Jones and Tertilt (2008)

Table 1: Fertility by Wealth Quintiles

Fertility rates: Total fertility rate						
Country	Survey Year	Household wealth index				
		Lowest	Second	Middle	Fourth	Highest
Sth. Africa	1998	4.8	3.6	2.7	2.2	1.9
Peru	1991-2	7	4.8	3.3	2.5	1.5
Brazil	1996	4.8	2.7	2.1	1.9	1.7
Pakistan	2006-7	5.8	4.5	4.1	3.4	3.0
Ukraine	2007	1.7	1.3	1.3	0.9	1

Source: (ICFI, 2012) http://statcompiler.com/?share=91398D8C3B

Given the near-linear relationship evident in the figure, equation (11) tracks the actual data quite well even as *Y* approaches its minimum value (normalized to zero).

The Malthusian Era:

Although there is plenty of evidence about the macro-relations between income levels and population growth rates at the level of international comparisons, until recently it was thought by many economic historians that that macro-relation had no analogue at the level of individual families.[12] We referred earlier to Adam Smith's observations about the importance of survival of infants born; a more extensive reference here may be useful. As Smith puts it, about his own time and place:

> "In some places, one half of the children die before they are four years of age; in many places before they are seven; and in almost all places before they are nine or ten. This great mortality will everywhere be found chiefly among the children of the common people who cannot afford to tend them with the same care as those of better station". [Smith, 1776; WN I.viii.38 p 97]

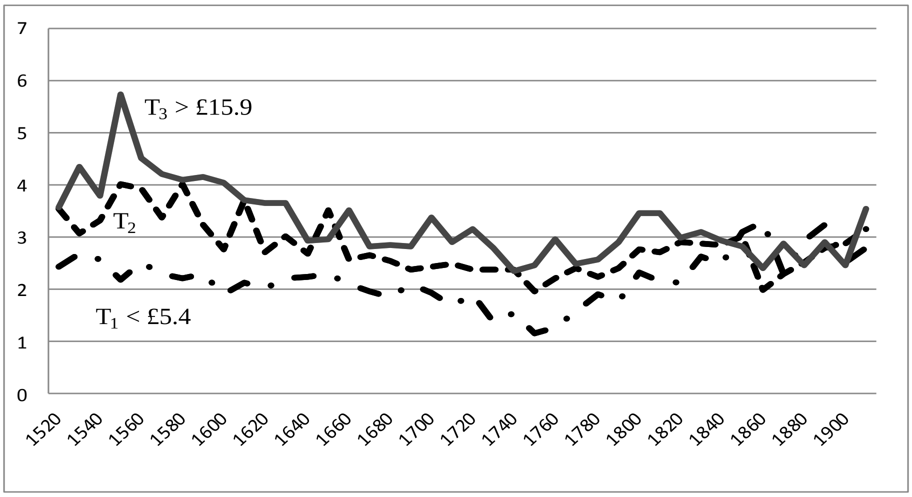
Thus it is the survival rate of children in the first decade of life rather than the birth rate as such that drives Smith's version of the Malthusian aggregate relation between rate of income growth and rate of population growth. But until recently, no direct empirical support for a positive relation between income and (surviving) family size has been available. Recently, however, Clark and Cummins (2015) have used wills from England over the 16th to 19th centuries to make a compelling case that there was a time when the within-economy relationship between income and the numbers of children was positive. They were able to construct estimates of lifetime earnings and numbers of surviving children for over 10,000 wills during a period when real incomes were broadly stable. Figure 2 below, which is close to Figure 7 in Clark and Cummins (2015), shows some striking trends.[13]

[12] With regards to England, "The limited and contradictory earlier evidence on the relationship between wealth and fertility in pre-industrial England, and the fact that marriage ages and nuptuality were seemingly similar in 1850 to their earlier levels of many decades, created a false impression that the fertility regime of the mid nineteenth century [the poor having at least as many children as the rich] represented the entire pre-industrial period." pg. 2, Clark and Cummins (2015, working paper version (2010) used for this and all subsequent refererences). See also their footnote 2.

[13] We are grateful for the scholarly and generous way in which they provided, and explained, their data to us.

Clark and Cummins defined income terciles in the figure as: T_1 below £5.4, T_2 between £5.4 and £15.9, and T_3 above £15.9[14]. They then divided the period 1520 to 1910 into 10-year sub periods and averaged the number of surviving children (at the time of the testator's death) for each income tercile. The figure suggests that from 1520 up to and including the 1780s, the number of children in the top income tercile was significantly higher than in lower terciles.[15] As we do, they call this the Malthusian era, but note that the relationship breaks down as England moved into the industrial revolution (which they date in the late 1700s). From that time, all income terciles show roughly the same numbers of children.

Figure 2: Surviving Children by Income Tercile



Source: Clark and Cummins (2015)

Since this is an important result in its own right and as indirect evidence of a systematic relationship between income and numbers of children, we have wanted to establish the econometric credentials of their conjectures from their original data. The relevant hypothesis

[14] It is sensible to define the terciles with the same figures over the whole period because income is stable. Only in the late C19th does this assumption start to weaken. The reader is referred to Clark and Cummins (2015) for a detailed discussion of their dataset. Although the sample size is very large (>10,000) it does not satisfy usual standards for random selection, given what can nowadays be accomplished by central statistical offices. It is, nevertheless, a remarkable and informative dataset.

[15] For the 1780s switch date, see Figure 8 of Clark and Cummins (2015).

is that the average number of children in each tercile differs in the Mathusian era in the manner suggested by Figure 2, but not afterwards. To that end we added a Malthus dummy *D* to Clark and Cummin's tercile dummies T_1, T_2 and T_3. The Malthus dummy is unity up to the 1780s and is zero from 1790. The full results are in Appendix 2. We confirm formally that there was indeed a period of history where income and numbers of surviving children were positively related. The appendix also estimates a pooled regression over the Malthusian period, corresponding to the Malthus dummy above and shown here as (12), which can be compared with (11) because income is normalized over (0,1).

$$N_t = 1.9947 + \underset{(t=14.34)}{1.5323}\, Y_t \qquad Y_t \in (0.1) \qquad R^2 = 0.0288,\ \sigma = 2.23 \tag{12}$$

Of course, this positive relationship reflects the inclusion of non-income factors that effect children. The rather large regression error standard deviation *(2.23)* suggests that Smith somewhat overstates the capacity of higher income households to protect their children; and the considerable variance in survival rates of children in households of identical size will serve to create greater dispersion of income in the next generation, effects that the Iron Law itself will have to overcome.

We do not of course have access to data that would allow our testing of the operation of the iron law directly in the Malthusian period. Nor have we attempted here to test the operation of the iron law in more modern times (for which reliable data on the time path of inequality is more readily available). That latter exercise will have to await a further occasion. All we have done here is to demonstrate a decisive shift in the sign of the income elasticity of numbers of children. It is this shift that motivates our title!

V Summary and Conclusions

There was a time when there was abroad, in the social world, an "invisible force" that made for greater equality in the distribution of income. If, as is likely, social and cultural forces biased our forebears towards unequal societies above their steady states, the invisible force would have held inequality on a short leash.

In this paper, we try to uncover that force, explain the logic of its operation and to derive some policy implications from it. The force in question is the connection between the income elasticity of family size in the current period and dispersion in the income distribution in the next. We describe this connection as an "Iron Law" because it makes appeal to what seem to us fairly robust assumptions the most significant of which is that a primary mechanism of income determination for individuals lies in the life-time intra-family transfers (including bequests where relevant) each individual receives. Those lifetime transfers include resources of sustenance, human capital acquisition, and protection from disease and death through the years of child dependence (which for most individuals over most of human history are far more significant than bequests received). But whether bequests or gifts inter vivos, in larger families the transfers received by each individual are smaller because of the larger the number of children among whom total family resources have to be shared. If family size is positively correlated with income/wealth, then the effects of this process across the whole economy is dispersion reducing. And if wealth plays a causal role in family size then disposition to have large families is effectively inheritable and so the process is inexorable, working its egalitarian magic relentlessly from generation to generation.

Things are not now as they were in the days of Adam Smith. Those same inexorable forces that over the centuries prior to Smith produced greater equality are now directed at producing greater inequality: the crucial parameter – the income elasticity of family size – has changed sign. The larger transfers received in upper income families are systematically divided among a smaller number of siblings (than in lower income families).

The implications of the iron law in the contemporary world for egalitarian policy await further empirical investigation. But one thing seems clear. Beyond the conventional array of egalitarian policies, attention should be given to mechanisms that block or moderate the effective size of intra-family transfers – such as gift and bequest taxation; and publicly funded education provision on an equal per capita basis. But in addition, the most distinctive suggestion that emerges here relates directly to the demographic aspects: greater consideration should be given to measures that increase the arithmetic value of the income elasticity of family size. The significance of this parameter for egalitarian policy has, we think, lain in obscurity for too long.

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Appendix 1: Scaling Distributions by Family Size

We begin with the observation of Blake (1989) that in a random sample of people, those from large families will be over-represented. For example, suppose that the pdf of family size is uniform over the integer support N=0 to 4. In generation 2 the distribution of adults classified by family-of-origin size will be 0.0, 0.1, 0.2, 0.3, 0.4 corresponding to N=0 to 4. This is obtained by multiplying each N by 1/5 and then dividing by the mean of the original pdf, namely 2 children, to ensure the probabilities sum to unity.

The question is to what extent that distorts the relationship between $V(Y_1)$ worked out with a generation *1* distribution of income and $V(n\,Y_1 / N)$ worked out with a generation *2* distribution, over-represented by children from large families. It turns out that the multiplication of Y_1 by n/N within the bracket exactly compensates for the scaling of the distribution.

Notation:

i	generations $i=1, 2$
f	pdf
Y	1st generation income (mean=y)
N	1st generation children (mean=n)
E_i	the expectation based on the pdf in generation i ($i=1, 2$)
V_i	the variance based on the pdf in generation i ($i=1,2$)

We intuit a relationship between $f_{NY,2}$ and the generation 1 density and then find the mean and variance of the new distribution. Whatever value of N is drawn for a family in generation 1, we assume the pdf will be scaled up by that value. Then, division by n is required so that the integral of the pdf is unity. This was the intuition above for Blake's observation that you are more likely to meet someone from a large family than a small one.

$$f_{NY,\,2} = \frac{N\,f_{NY,\,1}}{n}. \qquad (A1.1)$$

We are interested in $V_2(nY/N)$ – the variance of children's incomes calculated using the second generation distribution.

$$V_2\left(\frac{n\,Y}{N}\right) = E_2\left(\frac{n\,Y}{N}\right)^2 - \left(E_2\left(\frac{n\,Y}{N}\right)\right)^2 \qquad (A1.2)$$

The expectation of $n\,Y / N$ turns out to be the original expectation, namely y.

$$E_2\left(\frac{nY}{N}\right) = \int_N\int_Y \frac{n\,Y}{N}\frac{N\,f_{NY,1}}{n}\,dY\,dN = y \qquad (A1.3)$$

Equation (A1.3) shows how the multiplication by n/N addresses the problem of the scaled distribution. When evaluating $E_2\left(\frac{nY}{N}\right)^2$ in (A1.2) we first place it into the expectations integral, to find how it relates to an expectation based on a generation-1 distribution.

$$E_2\left(\frac{n\,Y}{N}\right)^2 = \int_N\int_Y \frac{n^2\,Y^2}{N^2}\frac{N\,f_{NY,1}}{n}\,dY\,dN = E_1\left(\frac{nY^2}{N}\right) \qquad (A1.4)$$

To evaluate the latter, we first do a Taylor series expansion of the term in the expectations.

$$\frac{nY^2}{N} \approx y^2 + 2y(Y-y) - \frac{y^2}{n}(N-n) + \frac{1}{2!}\left\{2(Y-y)^2 + \frac{2y^2}{n^2}(N-n)^2 - \frac{4y}{n}(Y-y)(N-n)\right\} \qquad (A1.5)$$

Upon taking generation-1 expectations, the second and third terms fall out.

$$E_1\left(\frac{nY^2}{N}\right) = y^2 + \frac{1}{2!}\left\{2E_1(Y-y)^2 + \frac{2y^2}{n^2}E_1(N-n)^2 - \frac{4y}{n}E_1(Y-y)(N-n)\right\} \qquad (A1.6)$$

When (A1.3) and (A1.6) are substituted into (A1.2) we obtain the equivalent expression to the one in the text.

$$
\begin{aligned}
V_2\left(\frac{n\,Y}{N}\right) &= E_2\left(\frac{n\,Y}{N}\right)^2 - \left(E_2\left(\frac{n\,Y}{N}\right)\right)^2 \\
&= E_1\left(\frac{nY^2}{N}\right) - y^2 \\
&= y^2 + \frac{1}{2!}\left\{2\,E_1(Y-y)^2 + \frac{2y^2}{n^2}E_1(N-n)^2 - \frac{4\,y}{n}E_1(Y-y)(N-n)\right\} - y^2 \\
&= V_1(Y) + \left(\frac{y}{n}\right)^2 V_1(N) - 2\left(\frac{y}{n}\right)C_1(Y,N)
\end{aligned}
$$

This confirms (7) as the basis for the difference equation linking inequality across generations.

Appendix 2: Confirmation of the Malthusian Era and Simulation Parameters

This appendix provides additional details that 1) demonstrate the existence of a Malthusian era and 2) provide a Malthusian era regression equation which was the basis for our simulations using income in section II. For the first task the basic regression (supressing the error term) is:

$$N=\gamma_1+\gamma_2 T_2+\gamma_3 T_3+\gamma_4 D+\gamma_5 D.T_2+\gamma_6 D\, T_3. \qquad \text{(A2.1)}$$

This can be rearranged using the fact that $\gamma_1= \gamma_1 (T_1+T_2+T_3)$ and $\gamma_4 D= \gamma_4 D\,(T_1+T_2+T_3)$ to give:

$$N = T_1\{\gamma_1+\gamma_4 D\}+ T_2\{\gamma_1+\gamma_2+(\gamma_4+\gamma_5) D\}+ T_3\{\gamma_1+\gamma_3+(\gamma_4+\gamma_6) D\}. \qquad \text{(A2.2)}$$

Table 2 shows a battery of tests applied to (A2.2). We run the regression both on the decade data of Figure 5 (cols. 2 and 3 below) and on the unit record data of every will (cols. 4 and 5). First, we test if the numbers of children in each tercile differ in the Malthusian era by setting $D=1$ in (A2.2) and taking the relevant paired differences (see table notes). The overwhelming conclusion in rows 1 to 3 of the hypothesis tests (see the bottom left of Table 2) is that the number of children in every income tercile differed significantly from the other terciles at the 1 per cent level, establishing a Malthusian (i.e. positive) covariance between Y and N up to and including the 1780s.

Table 2: Regression Results

(* and ** are two-sided significance for 5% and 1%. D=1 in Malthusian era. N_{Ti} is children in ith tercile)

		Decade Data		**Individual Data**		Notes
Var	Coeff	obs=120	(s.e.)	obs=12314	(s.e.)	Tercile dummy coefficient differences:
C	γ_1	**2.5862	0.15	**2.5382	0.08	$N_{T2}-N_{T1} = \gamma_2+ \gamma_5 D$
T_2	γ_2	0.1174	0.22	*0.2180	0.10	$N_{T3}-N_{T1} = \gamma_3+ \gamma_6 D$
T_3	γ_3	0.3603	0.22	**0.3580	0.10	$N_{T3}-N_{T2} =[\gamma_3- \gamma_2]+ [\gamma_6- \gamma_5] D$
D	γ_4	** - 0.5746	0.19	** - 0.5348	0.09	The differences in the Malthusian era
$D*T_2$	γ_5	**0.7538	0.26	**0.4640	0.12	compared to afterwards are respectively γ_5,
$D*T_3$	γ_6	**1.0485	0.26	**0.7904	0.12	γ_6 and $\gamma_6- \gamma_5$.
Hypothesis Tests: H_1 Coefficient Combinations for F-tests						Hypotheses
$\gamma_2+ \gamma_5>0$		**0.8712	0.15	**0.6820	0.06	$N_{T2}>N_{T1}$ in Malthusian era
$\gamma_3+ \gamma_6>0$		**1.4088	0.15	**1.1484	0.06	$N_{T3}>N_{T1}$ in Malthusian era
$[\gamma_3-\gamma_2]+[\gamma_6-\gamma_5]>0$		**0.5377	0.15	**0.4664	0.07	$N_{T3}>N_{T2}$ in Malthusian era
$\gamma_5>0$ as above		**0.7538	0.26	**0.4640	0.12	$N_{T2}-N_{T1}$ gap falls after Malthusian era
$\gamma_6>0$ as above		**1.0485	0.26	**0.7904	0.12	$N_{T3}-N_{T1}$ gap falls after Malthusian era
$\gamma_6-\gamma_5>0$		0.2947	0.26	**0.3264	0.11	$N_{T3}-N_{T2}$ gap falls after Malthusian era

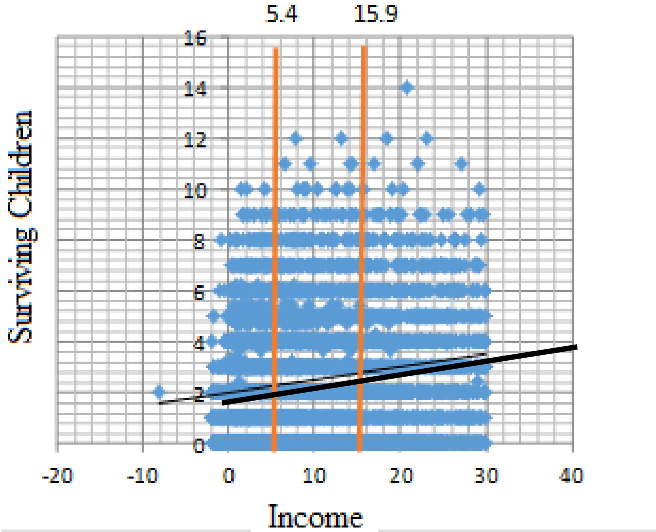
Notes: All tests are two sided, whereas the requirements of the different alternative hypotheses are one sided. However, when the estimated coefficients have the sign indicated under H_1, and H_0 is rejected with an F-test, these alternatives will certainly be rejected with a one sided t-test from an appropriately reparameterized regression, because the relevant p-value will halve.

Next, we tested whether this difference in children between the terciles was significantly greater in the Malthusian era compared with afterwards. We derived the tercile differences in the Malthusian era ($D=1$ in (A2.2)) and subtracted the same differences in the post-Malthusian era ($D=0$ in (A2.2)). We tested the significance of the relevant linear combinations of parameters. Here, the verdict from the individual data was that every tercile difference (N_{T2}-N_{T1}, N_{T3}-N_{T1} and N_{T3}-N_{T2}) fell significantly after the Malthusian era at the 1 per cent level. Apart from the difference N_{T3}-N_{T2}, the evidence from the decade data support the same conclusion.[16]

[16] Based on this dataset with Clark and Cummin's 1780s cut-off, there is weak evidence of a positive tercile difference in children numbers in the post-Malthusian era. That is, using the individual data and setting D=0 in Table 5, N_{T2}-N_{T1} (coefficient γ_2) is significant at the 5 per cent level, N_{T3}-N_{T1} (γ_3) is significant at the 1% level and N_{T3}-N_{T2} ($\gamma_3 - \gamma_2$, test not shown in Table 2) is not significant. However, the dataset contains only limited datapoints of the post-Malthusian era, and what significance there is almost certainly due to the ambiguity of

For our second task, we regressed the number of children on income, excluding income outliers. Naturally, the simulation regressions in the main text have two people added to these regressions to obtain family size.

Figure 6: A Malthusian Equation



The estimated equation in Table 4 has a highly significant slope coefficient, which is the marginal effect of income on family size. Since the maximum income (excluding the top 15% of income) was £30, we rescaled income to the interval [0, 1] and multiplied the coefficient by 30 to give the same marginal effect. The value of 1.55 says that the richest families (Y=1) had about one and a half more children than the poorest families (Y=0) during the Malthusian era.

Table 4: Malthusian Regression

Dependent Variable: N				
top 15% income excluded				
Sample (adjusted): 1 7060				
Variable	Coef	Std. Error	t-Statistic	Prob.
C	1.9947	0.0409	48.8281	0
Y (< £30)	0.0511	0.0036	14.2275	0
Rescaled Y ~ (0, 1)	1.5323	0.1077	14.2275	0
R-squared	0.0279	Mean dependent var		2.44
Adjusted R-squared	0.0277	S.D. dependent var		2.26
S.E. of regression	2.23			
F-statistic	205.5			
Prob(F-statistic)	0			

The R^2 of the regression for the Malthusian era is low, at around 2½ per cent, and the estimated error volatility σ is high, at just over two children. As noted earlier, the high error volatility, called entropy in the main text, is a welcome result because it saves the model from predicting vanishing inequality during the Malthusian era.

where to make the cutoff. Looking at Figure 5, it beggars belief that any differences would be significant after, say, the 1840s.