UTS Business School – Department of Economics
Dr Chau Chak Wing Building
14 – 28 Ultimo Road, Ultimo NSW 2007

T: +61 2 9514 7713
economics@uts.edu.au

Department of Economics

Working Paper Series

'A Short Period Sraffa-Keynes Model for the Evaluation of Monetary Policy'

Peter Docherty

University of Technology Sydney

No. 1 / 2021 **February 2021**

A Short Period Sraffa-Keynes Model for the Evaluation of Monetary Policy

Peter Docherty

Economics Group

UTS Business School,

University of Technology Sydney

Abstract

This paper develops a short period, one sector, Sraffa-Keynes model that can be used for the evaluation of various recommendations outlined in the Post Keynesian monetary policy literature. The model is characterised by the principle of effective demand, Sraffa or target-return pricing (which integrates the determination of key distributive variables and allows for short run cyclical variation in prices), conflict inflation, endogenous money, and a basic approach to monetary policy in the Smithin–Wray tradition of fixing the policy rate to achieve low or specified rates of unemployment. The paper argues that nominal interest rates are the appropriate target for monetary policy rather than real rates given the need to determine appropriate rates of return on capital and the good approximation that nominal rates are for the particular specification that real rates take in the model. A number of key results arise from model simulations: after experiencing two standard macroeconomic shocks, the model returns to a long period equilibrium characterised by the achievement of the target rate of return, desired capacity utilisation, and Sraffian prices of production. Monetary policy is also shown to operate through the typical Post Keynesian transmission mechanism of changes to income distribution. Flexible prices (where firms modify prices to cover the additional costs of running the capital stock at other than full capacity) are lastly shown to have similar effects on activity to monetary policy. Suggestions are made for further work which applies the model to the evaluation of counter-cyclical monetary policy, a comparison of fiscal and monetary policy responses to economic shocks, and which extends the model to a multi-sector context.

JEL Classification Numbers: E11, E12, E52.

Keywords*: monetary policy*, *demand shock*, *cost shock*, *income distribution*.

*Correspondence to Peter Docherty, Economics Group, UTS Business School, University of Technology Sydney, PO Box 123 Broadway, NSW, 2007, Australia. Ph. 61 2 9514-7780; Fax 61 2 9514-7777; E-mail: peter.docherty@uts.edu.au. Thanks to Tony Aspromourgos, Corrado DiGuilmi, Gordon Menzies, Rod O'Donnell and Prabhu Sivabalan for comments and suggestions.

1. Introduction

It is well known that mainstream economics reached a consensus prior to the *Global Financial Crisis* on the conduct of monetary policy (Goodfriend 2007). According to that consensus, monetary policy should bear the brunt of counter-cyclical macroeconomic management, focusing on inflation as its primary target, and adjusting the real interest rate in response to deviations of actual inflation from that target. Post Keynesians, by contrast, do not appear to have developed a similarly common mind on these issues. Different perspectives can be found in the Post Keynesian literature on whether monetary policy should target the real or the nominal interest rate, whether the policy rate should be fixed or varied counter-cyclically, and, if it is varied counter-cyclically, to which macro variables it should respond (Rochon 2007, pp.6-7). Moore (1994), for example, recommended varying interest rates counter-cyclically to reduce unemployment during downturns, while Fontana & Palacio-Vera (2006) accept the broad objective of targeting inflation (albeit within definite limits), and Palley (2007) advocates counter-cyclical variation of rates around what he calls the *minimum unemployment rate of inflation* (MURI). By contrast, Lavoie & Seccareccia (1999), Smithin (2007), Wray (2007) and Aspromourgos (2011) all advocate fixing interest rates permanently, and propose various criteria for deciding at what level this fixing should occur, from maximizing employment to more fairly distributing national income. In addition, Smithin (2007) and Aspromourgos (2011) argue that the real rate should constitute the policy target either because it is the relevant rate for expenditures or because it governs the real value of capital assets through time, while Wray (2007) asserts that central banks can only control *nominal* rates, and it is nominal rates that economic agents compare when making financial decisions.

Wray (2007, p.124) correctly observes that these Post Keynesian perspectives critically depend on the models that advocates have in mind in proposing them, and these models often remain implicit or lacking in detail. Different recommendations may thus be the result of different underlying models rather than differences in the perceived effects of policy actions. It would, therefore, be beneficial if the different Post Keynesian policy recommendations could be evaluated within the *same* theoretical framework. This would allow consistent evaluation based on an analysis of policy effects rather than only partially articulated presuppositions.

Of course this raises the question as to which model should be used for such evaluation, and at this point, differences between what Lavoie (2006, pp.89-90) calls American Post Keynesians, Kaleckians and Sraffians, not to mention other heterodox traditions including but not limited to Marxists and Institutionalists (*cf.* Aspromourgos 2004, p.191), might be raised. But the answer to this question is that a small range of well-articulated models could be used for this purpose. Such a strategy would provide a very rich heterodox assessment of policy actions because the effects of each proposal could be compared within each framework, and such comparisons then juxtaposed against similar comparisons within other frameworks. In conjunction with a good understanding of the limitations of formal models (see Dow 2013, p.79), similar conclusions reached across models would take on particular significance, and differences in evaluations could be understood in the light of each model's distinctive features.

This paper seeks to strengthen this area of Post Keynesian research by developing a short-run model in the Sraffa-Keynes tradition.[1] This tradition provides a carefully articulated theoretical structure, the components of which are well-suited to thinking about monetary policy. Income distribution (which Rochon & Setterfield, 2008, pp.15, 23, suggest is the agreed primary transmission mechanism of monetary policy across the Post Keynesian literature), price determination (particularly relative price determination), output and employment levels, and the relationship between monetary variables and the real economy, have been four key areas of research within this framework (Aspromourgos 2004, pp.182-185), and each of these is important for understanding the impact of monetary policy. It is true, however, that models in this tradition have tended to be long period in nature, and a short run variant is needed to consider the full range of monetary policy approaches identified in the Post Keynesian literature.

The paper is structured as follows. Section 2 considers a number of issues associated with a short-run version of the Sraffa-Keynes model. Sections 3 to 5 outline various dimensions of the model: its output and employment characteristics; its distributional characteristics; and its approach to inflation. Section 6 derives the model's reduced form, equilibrium and stability properties, and the model is calibrated in Section 7. A number of standard macroeconomic shocks are simulated in Section 8 before conclusions are drawn in Section 9.

[1] The assumption is made here that the Sraffian tradition falls within the classification of Post Keynesian economics. While the literature contains some support for this classification (see Lavoie 2006, pp.89-91), it is recognised that others are less comfortable with it. King (2015, pp.114-117) analyses potential reasons for such discomfort in terms of the long period focus of Sraffian analysis in contrast to the short period focus of Kaleckian and other models, and their tendency to downplay the possibility that the long run has any unique analytical character. Aspromourgos (2004, p.181f) suggests that there is not even a single "Sraffian School" but rather a set of research programmes which share common features derived from Sraffa's interpretation of the Classical surplus approach. The intention in the present paper is not to contribute to this debate but simply to accept the various theoretical perspectives that Sraffian models (especially those associated with the so-called Sraffa-Keynes synthesis) have in common with Kaleckian and other heterodox traditions, and to use the term "Post Keynesian" to express those commonalities. The underlying premise of the paper is designed to at least recognise the points of difference identified by King. As suggested above, the model offered is viewed as only one in a suite of models that might be used for the evaluation of monetary policy, and Dow's (2013) reminder that any model is simply an abstract representation of a more complex reality is accepted, so that models must be combined with non-formalist perspectives to draw reasonable conclusions about the world.

2. The Nature of a Short Period Sraffa-Keynes Model

Models in the Sraffa tradition focus on the relationship between distribution of the national product and "normal prices" or so-called "prices of production" (Aspromourgos 2004, p.181). These prices reflect the "persistent", "non-temporary" or "non-accidental" forces operating within the economic system, and are thus assumed to be long period forces (Kurz & Salvadori 1995, pp.1-2). But as Aspromourgos (2004, p.182) observes, because such models focus on the specific issue of the relationship between distribution and prices, they are compatible with a range of explanations for other economic phenomenon such as the determination of output and employment. One long-standing research programme has thus investigated the possibility of marrying the Sraffian determination of prices and distribution with Keynes's explanation of output and employment (see Eatwell & Milgate 1983). Models within this so-called *Sraffa-Keynes synthesis* have, however, tended to retain their long period focus, arguing that Keynes's principle of effective demand is capable of playing a role in determining economic growth over the long run. Garegnani (1992), Serrano (1995), Cesaratto, Serrano & Stirati (2003) among others, all adopt such a perspective.

One group of models within this Sraffa-Keynes synthesis that has explored the behaviour of the system outside of long period equilibrium has focused on the possibility that disequilibrium prices might deviate from their long period normal values and reflect short run imbalances between supply and demand (see, for example, White 1998). Dumenil & Levy (1999) have also explored the path that a Sraffa-Keynes type economy might take from the short run in which it is dominated by Keynesian dynamics to the long run in which it is dominated by Classical dynamics. Both such approaches, however, ask the legitimate question of what a Sraffa-Keynes economy might look like in the short period along the path to its long period position.

An additional perspective of relevance to the investigation of monetary policy has been the exploration of a "monetary closure" for the Sraffian system (Sraffa 1960, p.33) whereby it is the rate of profit that is determined by the rate of interest rather than the other way around. In this case, the real wage and $n-1$ relative prices in an economy with n commodities are endogenous to the Sraffian pricing system (see Pivetti, 1985; and Panico, 1988). Such an approach would also appear compatible with the Post Keynesian theory of endogenous money according to which monetary rates of interest are under central bank control (Kaldor 1986; Moore 1988; and Cottrell 1994, pp.596-601). Thus, if the rate of interest is continually being adjusted by the central bank because an active approach is being taken to monetary policy, there are likely to be implications for distribution in a Sraffa-Keynes model, and unpacking the short period from the long period implications of this influence is a legitimate endeavour.

In general then, asking how a Sraffa-Keynes model behaves in the short period, and exploring what the path from short period to long period looks like, is not simply a reasonable project but an important one in the development of the Sraffa-Keynes synthesis. Using such a model for the exploration of monetary policy is also important given the dual possibilities of central bank determination of interest rates in the short period and long period monetary closure in Sraffa-Keynes models.

This investigation does, however, require careful definition of the terms "short period" and "long period". Sanfilippo (2011) has observed some of the ambiguities associated with this distinction and suggests replacing this terminology with a careful specification of assumptions about which factors are held constant and which are allowed to vary over what time frames in any given model. We adopt a half way approach to this suggestion. We retain the two terms but we carefully define how the term *long period* will be understood, specify the meaning of the term *short period* in relation to the long period, and then make as clear as possible which variables are allowed to vary over which time frame within the model.

There is general agreement in the Sraffian literature that the long period is characterised by prices that are equal to their "normal" values (and thus constitute "prices of production"), a uniform rate of profit earned on capital across the economy, and capacity utilisation equal to its "normal" or "desired" rate (see Milgate 1982, p.12; Dumenil & Levy 1999, p.687; Lavoie 2003, p.59; and Sanfilippo 2011, p.372). [2] The long period also tends to be the analytical time frame within which the issue of growth is addressed (Sanfilippo 2011, p.372).

This approach to defining the long period will be adopted here and the features described above will be used to identify "equilibrium" in the analysis to follow. We will also add the condition that the actual rate of profit should equal the target or desired rate of profit since the possibility of these being different will characterise our model. We will also simplify the analysis slightly in order to focus attention on the short period by assuming that the long period growth rate of the economy is zero. Later, we will show the precise point at which this assumption affects the analysis. All of this implies that the economy is in the *short period* when the characteristics identified above *do not* hold. In such circumstances, the economy will not be in equilibrium and will be in a process of change and adjustment. There is, therefore, no

[2] Freitas & Serrano (2015) distinguish between *long period positions* and *fully adjusted position*s in which capacity utilisation is equal to its "desired" level. It is thus possible for long period positions not to be characterised by equality between actual and desired capacity utilisation in their approach.

such thing as "short period equilibrium" in this model. But there are short period relationships, and because we employ a discrete time framework where each time period will be thought of as roughly equivalent to one quarter (three months) in real time, there is a sense in which a sequence of short period "positions" of the economy will lead into the long period (Dumenil & Levy 1999, p.688). We are not claiming that it never makes sense to think in terms of short period equilibrium, we are simply choosing to model the relationship between short period and long period in this piece of analysis so that only the latter is understood in terms of "equilibrium". We also reiterate our acceptance of Dow's (2013) insistence that the limitations of formal modelling should be kept in mind when using approaches such as those used in this paper, and that the results of formal models be combined with qualitative judgement when drawing conclusions.

The model is outlined in Sections 3 to 6 and once its structure has been considered, its reduced form is derived, equilibrium values of the variables in this form are obtained, and the model's stability characteristics are explored. The model is then calibrated in order to simulate the kinds of shock that may be relevant to the operation of monetary policy. The Australian economy has been chosen against which to calibrate the model since the author is more familiar with that economy than with any other. In principle, however, any economy could be chosen for this calibration exercise. Two standard types of macroeconomic shock are then used to investigate the model's dynamics: a negative demand shock; and a positive cost shock with the potential to increase inflation. In each of these cases, baseline trajectories for key variables are compared to their trajectories under alternative specifications of the model in order to explore the significance of particular model features (such as different approaches to pricing, for example) or different specifications for the core approach to monetary policy.

3. Output and Employment in the Sraffa-Keynes Model

The model developed over the next three sections is in the spirit of Aspromourgos (1991), Kurz & Salvadori (1995), White (1998) and Docherty (2012) which has at its heart a combination of the principle of effective demand and Sraffa or target-return pricing which integrates the determination of key distributive variables. It draws on the Kaldor-Pasinetti tradition to link these features of the model by making aggregate demand sensitive to the distribution of income, it incorporates money supply endogeneity from Kaldor (1986) and Moore (1988), and inflation is interpreted as a conflict over income distribution following Rowthorn (1977). The model will have only a single productive sector (Kurz & Salvadori 1995, pp.42-57) that allows us to focus on the channels through which monetary and real aspects of the model affect each other.

It is clear that such an approach ignores important sectoral interactions that may affect macroeconomic outcomes but we leave expansion of the model to deal with such sectoral interactions to future work.

The output and employment features of our one sector model are outlined in equations (1) to (7) below. The central Keynesian feature of the model is reflected in equation (1) which is simply the *principle of effective demand* where the economy's aggregate output, Y_t, is determined contemporaneously by the standard components of aggregate demand: consumption spending, C_t, investment spending by firms, I_t, autonomous government spending, $\bar{G}$, and autonomous net exports, $\overline{NX}$. Consumption spending is determined in equation (2) and reflects the Kaldor (1955-56)-Pasinetti (1974) principle that spending is a function of income distribution. Households are thus divided into two types: those whose income is derived from selling their labour and who thus earn wage income in each period, W_t; and those who own the means of production and whose income is derived from business profits in each period, P_t. These households have different savings propensities as in the standard Kaldor-Pasinetti set-up where s_w and s_c are the savings propensities of worker and capitalist households respectively. We also assume that all households are taxed at the same rate of τ. Consumption in equation (2) is, therefore, made up of a single autonomous component, $\bar{C}$, a component equal to a proportion of after-tax wage income from the previous period, $(1 - s_w) \cdot (1 - \tau) \cdot W_{t-1}$, and a component equal to a proportion of after-tax profit from the previous period, $(1 - s_c) \cdot (1 - \tau) \cdot P_{t-1}$.

$$Y_t = C_t + I_t + \bar{G} + \overline{NX} \tag{1}$$

$$C_t = \bar{C} + (1 - s_w) \cdot (1 - \tau) \cdot W_{t-1} + (1 - s_c) \cdot (1 - \tau) \cdot P_{t-1} \tag{2}$$

$$I_t = \delta \cdot \nu \cdot Y_{t-1} \cdot u_{t-1} \tag{3}$$

$$u_t = \nu \cdot Y_t / K_t \tag{4}$$

$$K_t = K_{t-1} \cdot (1 - \delta \cdot u_{t-1}) + I_{t-1} \tag{5}$$

$$UE_t = (\bar{N} - \ell_t \cdot Y_t)/\bar{N} \tag{6}$$

$$\ell_t = (1 - \beta) \cdot \ell_{t-1} \tag{7}$$

Investment spending by firms, given in equation (3), takes a Kaleckian-Steindlian form with variations (Dumenil & Levy 1999, p.691). Investment is influenced positively by the rate of capacity utilisation, u_t (though with a lag), but the constant term is replaced with an accelerationist component, $\delta \cdot \nu \cdot Y_{t-1}$, where δ is the rate at which fixed capital physically depreciates, and ν is the technical capital-output ratio. We define this capital-output ratio with respect to potential output so that $\nu = K_t/Y^P$ where K_t is the capital stock at time t and Y^P is potential output, the conventional maximum that can be produced with that level of capital.[3] For the sake of simplicity, desired capacity utilisation will be assumed equal to unity.[4] The multiplicative form of equation (3) thus implies that in equilibrium with $u_{t-1} = 1$ and output equal to Y^* (i.e. $Y_{t-1} = Y^* = Y^P$), investment simply replaces worn out capital from the previous period, leaving the capital stock constant through time.[5]

The capacity utilisation rate is determined in the conventional manner by equation (4) as the amount of capital technically required to produce output in the current period ($\nu \cdot Y_t$) divided by the existing capital stock in that period, K_t. Substitution of the definition for ν generates the alternative definition that $u_t = Y_t/Y^P$ (*cf.* White 1996, p.287). The capital stock is determined in conventional fashion by equation (5) except that only capital *actually used* to produce output in a particular period (as opposed to that which remains idle) is assumed to physically depreciate, so that the depreciation rate is multiplied by the capacity utilisation rate to determine worn out capital from the previous period.

Equation (6) is simply a definition of the unemployment rate UE_t. It specifies this rate as the number of unemployed workers divided by the exogenously given workforce, $\bar{N}$. The number of unemployed workers is itself given by the difference between the exogenously given workforce and total employment, $\ell_t \cdot Y_t$, where ℓ_t is the technically given unit labour requirement for production or the labour-output ratio in period t. Equation (7) determines this labour requirement as that from the previous period less a proportion determined by any exogenous change in labour productivity, β, where $0 < \beta < 1$. As discussed further below, most of the time we will assume that $\beta = 0$ since we are abstracting from growth. But we will be interested in one-off changes to productivity, and in this case we will allow $\beta > 0$.

---

[3] This may differ from the level of output necessary to employ all of the available workforce.

[4] This ignores the issue of firms choosing a normal rate of capacity utilisation that is profit maximising (see White 1996, 1998). The normal rate of capacity utilisation is simply treated as a convention in this model set equal to one for the sake of simplicity. We assume that this does not rule out running the capital stock above its conventional capacity so that the capacity utilisation rate can exceed 1 for short periods of time. But this will have two effects in the present model: it will lead to additional investment according to equation (3) which will expand capacity over time; and it will increase costs and reduce profits according to a pricing equation developed below. It should also be emphasised that equation (3) does *not* suggest that firms are spending out of the circular flow of income, so that $\delta \cdot \nu$ could be understood as a *marginal propensity to invest* (as, for example, Freitas & Serrano 2015, p.261 appear to do). Investment may thus require external finance although we abstract from this issue.

[5] An alternative formulation of the investment function might make investment spending responsive to the actual rate of profit earned relative to the target rate of profit, for example: $I_t = \delta \cdot \nu \cdot Y_{t-1} \cdot \left[1 + q \cdot \left(r^a_{n,t} - r^*_{n,t}\right)\right]$, where $r^a_{n,t}$ is the actual rate of profit in a given period, $r^*_{n,t}$ is the desired or target rate of profit in any period, and q is an adjustment parameter. We tested this formulation and it made no fundamental difference to equilibrium, stability or the general shape of the time paths of any variable.

4. Distribution and Prices

The price and distribution system for our model is outlined by equations (9) to (14). To see how these equations were obtained, consider the *actual* or *realised* rate of profit in period *t*, $r^a_{n,t}$, given by equation (8) below:

$$r^a_{n,t} = \frac{p_t \cdot Y_t - MW_t \cdot N_t - p_t \cdot \delta \cdot \nu \cdot Y_t - Z \cdot (Y_t - Y^P)}{p_t \cdot K_t} \tag{8}$$

We assume that the price of goods and services, p_t, is set by firms at the beginning of each period for sales that occur during the period. Total revenue is thus determined by multiplying output and sales during the period by this price to obtain p_tY_t. The real cost of capital depreciation during the period, $\delta \cdot \nu \cdot Y_t$, can similarly be valued at this price, and can thus be expressed as $p_t(\delta \cdot \nu \cdot Y_t)$. We also assume that labour costs are given by a money wage set at the beginning of the period, MW_t, multiplied by employment during the period, N_t, to give the total wage bill as MW_tN_t. We assume that there are additional costs associated with running the capital stock at greater than full capacity which are given by the product of a unit dollar cost Z, and the output gap, $(Y_t - Y^P)$. This formulation implies that there are also *savings* from running the capital stock at *below* full capacity which we accept for the sake of simplicity. The rate of profit is thus given by total revenue minus total costs as set out in the numerator of the above expression divided by the value of capital invested at the beginning of the period. We can express the amount of this capital as K_t but it must be valued to obtain the money rate of profit earned by firms. The question is whether we use the price set by firms at the beginning of the period, p_t, or the price set in the *previous* period, p_{t-1}, to perform this function.

The argument for using p_{t-1} is that this was the prevailing price when the capital stock was purchased and put in place to generate the cash flows specified in the numerator of the rate of profit expression above. The case for using p_t is that firms have the option of selling their capital at the beginning of the period at this price, and investing the proceeds in an alternative asset. In order to obtain the appropriate rate of return on the capital invested in the production process, therefore, that capital must be valued at the most up to date price, and this is p_t. The choice of p_{t-1} or p_t at this point has implications for whether it is the real or the nominal rate of profit that appears in the wage-profit frontier to express a relation between this rate and the real wage. This will have follow-on implications for the central bank's choice of real versus nominal interest rates for monetary policy which we saw earlier was a point of difference between Smithin and Wray in the Post Keynesian monetary policy literature.

$$r_{n,t}^{a} = u_t \cdot [\tfrac{1}{v} \cdot (1 - w_t \cdot \ell_t) - \delta] - \tfrac{z_t}{v} \cdot (u_t - 1) \quad (9)$$

$$w_t = \tfrac{1}{\ell_t} \cdot [1 - v \cdot (r_{n,t}^{*} + \delta) - z_t \cdot (1 - 1/u_t)] \quad (10)$$

$$r_{n,t}^{*} = [\textstyle\sum_{j=1}^{n} i_{WL,t-j}] + \sigma_k \quad (11)$$

$$i_{WL,t} = i^{*} \quad (12)$$

$$W_t = w_t \cdot \ell_t \cdot Y_t \quad (13)$$

$$P_t = Y_t - W_t - \delta \cdot v \cdot Y_t \quad (14)$$

We elect for p_t in this choice for two reasons. The first is that since determination of appropriate rates of return on capital is central to the current analysis, valuing capital in terms of its nearest alternatives suggests the need for current prices rather than historical prices. Secondly, the choice of p_t implies that the wage-profit frontier will contain the nominal rate of profit and this may actually be a good approximation for the form that the real rate of profit would have taken had p_{t-1} been used instead. [6]

Equation (8) will not be a very convenient form in which to express the rate of profit for the analysis that follows. We can, however, transform equation (8) by breaking up its R.H.S. into a number of component parts as follows:

$$r_{n,t}^{a} = \frac{Y_t}{K_t} - \frac{MW_t \cdot N_t}{p_t \cdot K_t} - \delta \cdot v \cdot \frac{Y_t}{K_t} - \frac{Z \cdot Y_t}{p_t \cdot K_t} - \frac{Z \cdot Y^P}{p_t \cdot K_t} \quad (15)$$

Then, noting that $Y_t/K_t = u_t/v$, $N_t/K_t = (\ell_t/v) \cdot u_t$, $v = K_t/Y^P$, [7] and that the real wage, w_t, is given by MW_t/p_t, allows us to express (15) as:

$$r_{n,t}^{a} = u_t \cdot \left[\frac{1}{v} \cdot (1 - w_t \cdot \ell_t) - \delta\right] - \frac{Z}{p_t} \cdot \frac{1}{v} \cdot (u_t - 1) \quad (16)$$

Since Z represents the unit cost of operating the capital stock at other than normal capacity, we may treat Z/p_t as the associated real cost of this operation and represent it with the symbol z_t. Substituting this into (16) gives:

[6] This is shown in the Appendix.

[7] That $Y_t/K_t = u_t/v$ follows from the fact that we may express Y_t/K_t as $(Y_t/K_t) \cdot (Y^P/Y^P)$. This product may be rearranged to give: $(Y_t/Y^P) \cdot (Y^P/K_t)$. This first bracketed term was shown above to equal the rate of capacity utilisation, u_t, and the second is simply the inverse of the technical capital-output ratio, v. Thus: $Y_t/K_t = u_t/v$. That $N_t/K_t = (\ell/v) \cdot u_t$ follows from the fact that we can express N_t/K_t as $(N_t/K_t) \cdot (Y_t/Y_t)$. This can be rearranged to give: $(N_t/Y_t) \cdot (Y_t/K_t)$. The first of these bracketed terms is simply the unit labour requirement for production, ℓ, while we have just shown that the second term is u_t/v. It thus follows that $N_t/K_t = (\ell/v) \cdot u_t$.

$$r_{n,t}^{a} = u_t \cdot \left[\frac{1}{v} \cdot (1 - w_t \cdot \ell_t) - \delta\right] - \frac{z_t}{v} \cdot (u_t - 1)$$

This is just equation (9) above. It should be noted that this rate will vary across the cycle as capacity utilisation differs from unity. When, however, capacity utilisation is at its desired level of unity, the rate of profit will equal its long run or fully adjusted rate (Lavoie 2003, p.45) which we will designate as $r_{n,t}^{*}$. This will be given as follows:

$$r_{n,t}^{*} = \frac{1}{v} \cdot (1 - w_t \cdot \ell_t) - \delta \tag{17}$$

Thus in general, we may also express the actual or realised rate of profit in any period as:

$$r_{n,t}^{a} = u_t \cdot r_{n,t}^{*} - \frac{z_t}{v} \cdot (u_t - 1) \tag{18}$$

We may also use equation (8) above to derive a price-setting equation for firms. Multiplying both sides of (8) by p_t, breaking up the R.H.S. of the resulting equation into its component parts and noting that $Y_t/K_t = u_t/v$, $N_t/K_t = (\ell_t/v) \cdot u_t$, and $v = K_t/Y^P$ as we did above, gives:

$$p_t \cdot r_{n,t}^{a} = p_t \cdot \frac{u_t}{v} - MW_t \cdot \frac{\ell_t \cdot u_t}{v} - p_t \cdot \delta \cdot u_t - p_t \cdot \frac{z_t}{v} \cdot (u_t - 1) \tag{19}$$

Collecting all terms containing p_t on the L.H.S., multiplying by –1, and factorising gives:

$$p_t \cdot [\frac{u_t}{v} - \delta \cdot u_t - \frac{z_t}{v}(u_t - 1) - r_{n,t}^{a}] = MW_t \cdot \frac{\ell_t \cdot u_t}{v} \tag{20}$$

Dividing by the term in square brackets on the L.H.S. and rearranging gives an expression for the price level:

$$p_t = \left[\frac{\ell_t}{(1 - \delta \cdot v - r_{n,t}^{a} \cdot v/u_t) - z_t \cdot (1 - 1/u_t)}\right] \cdot MW_t \tag{21}$$

Equation (21) expresses the relationship between the unit price level, the actual rate of profit, wage costs, the technical conditions of production, and the rate of capacity utilization. But firms may also use this equation as a basis for institutional price-*setting*. If we assume that firms in our model set prices to achieve a long run target rate of return (Lee 1994, p.310), $r_{n,t}^{*}$, but also to cover the additional costs of running the capital stock at anything other than normal capacity (in our model equal to unity), then the price level in each period is given by:

$$p_t = \left[\frac{\ell_t}{[1-\nu\cdot(r^*_{n,t}+\delta)]-z_t\cdot(1-1/u_t)} \right] \cdot MW_t \qquad (22)[8]$$

This equation determines the unit price level as a mark-up over money wage costs where the mark-up is multiplicative rather than additive, and also depends on the technical requirements of production, the firm's target rate of profit, and the rate of capacity utilisation (*cf.* Lavoie 2003, p.57). In the long run or fully adjusted position with $u_t = 1$, the $z_t \cdot (1 - 1/u_t)$ term will be zero, and the prices given by equation (22) will be those consistent with the long run rate of return, $r^*_{n,t}$. These prices will thus constitute *long period* prices, *normal* prices, or *prices of production* in the Sraffian sense (White 1996, pp.281-282; *cf.* Lavoie 2003, p.57).

Equation (22) will, however, remain implicit in our model and we will use it to express the real wage in each period. Moving MW_t to the L.H.S. and inverting both sides gives us the following expression:

$$w_t = \frac{1}{\ell_t} \cdot [1 - \nu \cdot \left(r^*_{n,t} + \delta\right) - z_t \cdot (1 - 1/u_t)]$$

Which is just equation (10) in the model above. As u_t varies through the cycle, with a given money wage, MW_t, the real wage, w_t, will also vary. In long period equilibrium, however, with $u_t = 1$, equation (10) reduces to:

$$w_t = \frac{1}{\ell_t} \cdot [1 - \nu \cdot \left(r^*_{n,t} + \delta\right)] \qquad (23)$$

which is the standard wage-profit frontier in a one commodity Sraffian model (see, for example, Pasinetti 1977, pp.112-115, Aspromourgos 1991, pp.108-109, and Kurz & Salvadori 1995, p.46). Note, however, that this relation contains the nominal rate of profit, $r^*_{n,t}$, as a result of the earlier choice of p_t rather than p_{t-1} in equation (8).

The causality structure with respect to distribution and prices thus runs as follows in our model. Firms formulate the target rate of return on capital using equation (11). According to this equation, firms take an average of the central bank-determined policy rate over the previous *n* periods and add an adjustment for risk and trouble, σ_k reflecting Sraffa's (1960, p.33) suggestion for closure of the pricing and distribution system. This adjustment could be determined in a more sophisticated way by taking account of variation in actual profits over

---

[8] Note that if we exclude the additional costs of running the capital stock at anything other than normal capacity, it can be shown that $r^a_{n,t} = u_t \cdot r^*_{n,t}$. The rate of profit adjusted by the rate of capacity utilisation in equation (21) would then simply be equal to the target rate of profit. It thus constitutes a reasonable pricing strategy for firms to replace $r^a_{n,t}/u_t$ in equation (21) with $r^*_{n,t}$, and to rely on the $z_t \cdot (1 - 1/u_t)$ term to account for additional costs associated with the business cycle. This is precisely what equation (22) above does.

some backward-looking reference period, but in this model we assume it to be constant, depending on the attitude of firm owners to risk.

This target rate of profit is fed into equation (22) in conjunction with the rate of capacity utilisation to determine prices. Price determination, given the money wage, also determines the real wage in each period according to equation (10). Once the real wage is determined, this feeds into equation (9), along with the rate of capacity utilisation, to determine the *actual* or *realised* rate of profit in that period. In the long run, however, it will be true that $u_t = 1$, so that the *actual* rate of profit will equal the *target* rate of profit, and prices will constitute *normal* prices in the Sraffian sense. We show later in the paper that the condition $u_t = 1$ is met in long run equilibrium.

The logic of equation (12) follows from our preference for incorporating the nominal rate of profit into the wage-profit frontier. We assume that the central bank chooses a target value for the nominal policy rate, $i*$, and sets the nominal rate at this level. This then affects the target rate of profit chosen by firms via equation (11). Such an approach is consistent with Wray's (2007) argument that nominal rates are appropriate monetary policy instruments in Post Keynesian models and that the best policy is to set the nominal rate at zero. In contrast to this, we saw above that Smithin (2007) recommended that it should be the *real* rate that is set at a low and stable value in order to promote full employment. In our model, it would be possible for the authorities to set a target for the unemployment rate and to solve the model for the policy rate that delivers this level of unemployment. But this rate would be the nominal rather than the real rate, and it would not necessarily be zero, depending on the structure of the model. Were that structure to change (due say to a change in autonomous expenditures) the employment-target policy rate would also change. This approach thus incorporates aspects of both the Smithin and Wray approaches to monetary policy and we call it the *nominal employment target* approach. It constitutes one interpretation of equation (12). An alternative interpretation is a *pseudo-Wray* rule where the nominal policy rate is set at some low level (close to but not necessarily zero) and left at that level. We call this the *constant nominal rate* approach and in the simulations reported in Section 7 below, we compare both approaches to understanding equation (12).

The core determination of income distribution within the Sraffa-Keynes model is thus given by equations (9) to (12). Equation (13) determines aggregate real wages as the product of the real wage rate multiplied by the level of employment as explained above. Aggregate profits are determined in (14) as the residual from national income after deducting aggregate real wages and real depreciation.

5. Inflation

The model's inflation dynamics are given by equations (24) and (25) which we interpret in terms of Rowthorn's (1977, pp.216-219) model of conflict inflation. Rochon & Setterfield (2007, p.28; *cf*. Lavoie 2003, pp.65-67) specify a version of this approach in terms of two equations that determine nominal wage inflation and price inflation respectively. In Rochon and Setterfield's approach, nominal wage inflation is a function of Rowthorn's (1977, p.217) "aspiration gap" where this is specified in terms of the difference between workers' desired wage share and the actual wage share, as well as of expected inflation and growth in labour productivity. Inflation is a function of the difference between the actual wage share and the wage share firms regard as reasonable (which Rochon and Setterfield assume to be less than its actual value), and the difference between nominal wage growth and growth in labour productivity, reflecting downward pressure on profits.

$$\pi_t = h_1 \cdot \pi_t^e + (\alpha - \beta) - h_2 \cdot UE_{t-1} \tag{24}$$

$$\pi_t^e = \pi_{t-1}^e + h_3 \cdot (\pi_{t-1} - \pi_{t-1}^e) \tag{25}$$

We employ a similar logic but specify this in terms of a standard Phillips Curve analysis. Equation (24) is an expectations-augmented Phillips Curve which differs from Friedman's (1968) version of the curve in that the coefficient on expected inflation is assumed to be less than unity, so that the value for unemployment achieved when inflation expectations are realised is not a constant but varies inversely with the level of unemployment (see Gordon 2011, p.18). This formulation is derived from our approach to pricing in equation (22) above. If we let $B_t = (1 - \nu \cdot (r_{n,t}^* + \delta) - z_t \cdot (1 - 1/u_t)$, then we can express equation (22) as:

$$p_t = \frac{\ell_t \cdot MW_t}{B_t} \tag{26}$$

In this case, the percentage change in p_t will be given as follows:

$$\frac{\Delta p_t}{p_t} = \frac{\Delta \ell_t}{\ell_t} + \frac{\Delta MW_t}{MW_t} - \frac{\Delta B_t}{B_t} \tag{27}$$

Note that we may define labour productivity as the inverse of the unit labour requirement and designate the percentage increase in labour productivity in any period as $\beta_t = -(\Delta \ell_t / \ell_t)$. If we also assume that $\nu, r_{n,t}^*$ and δ are all constant, and that the real cost of operating the capital stock at other than full capacity, z_t, is also constant, so that $z_t = \bar{z}$, then it is easy to show that:

$$\frac{\Delta B_t}{B_t} = \frac{\bar{z} \cdot \Delta u_t^{-1}}{1 - \nu \cdot (r_{n,t}^* + \delta) - z_t \cdot (1 - 1/u_t)} \tag{28}$$

Defining inflation as $\pi_t = \Delta p_t / p_t$ then allows us to write equation (27) as:

$$\pi_t = \frac{\Delta MW_t}{MW_t} + \frac{\bar{z} \cdot \Delta u_t^{-1}}{1 - \nu \cdot \left(r_{n,t}^* + \delta\right) - \bar{z} \cdot (1 - 1/u_t)} - \beta_t \tag{29}$$

Inflation is thus driven by the sum of percentage increases in money wages and the costs associated with operating the capital stock at other than full capacity, less any increase in labour productivity.

The percentage increase in money wages is given by equation (30):

$$\Delta MW_t / MW_t = \vartheta_1 \cdot [\pi_t^e + \psi - \vartheta_2 UE_t] \tag{30}$$

This asserts the familiar argument that workers formulate their target increase in money wages based on three factors. The first is expected inflation, π_t^e, reflecting a concern with the real value of wages. The second is an overall aspiration to increase wages by a factor of ψ. The third affects the strength with which workers pursue a money wage increase based on the first two factors, and depends on how workers perceive their bargaining power. We assume that this is a function of the rate of unemployment, so that this factor is given by $\vartheta_2 UE_t$, where ϑ_2 is a parameter reflecting how unemployment affects perceived bargaining power. The higher is the rate of unemployment, the lower, we assume, is the perceived bargaining power of workers in wage negotiations. The overall target rate of money wage increases is thus given in the square brackets on the R.H.S. of equation (30). But the actual increase in money wages is affected by a series of other factors apart from this target wage increase. These factors include the actual bargaining strength of workers, institutional factors such as the role and power of wage tribunals or regulatory commissions, and whether the overarching political climate favours labour or firm owners in wage setting processes. The effect of these factors is captured by parameter ϑ_1.

Substituting equation (30) into (29), and expanding gives the following:

$$\pi_t = \vartheta_1 \cdot \pi_t^e + \vartheta_1 \cdot \psi - \vartheta_1 \cdot \vartheta_2 UE_t + \frac{\bar{z} \cdot \Delta u_t^{-1}}{1 - \nu \cdot \left(r_{n,t}^* + \delta\right) - \bar{z} \cdot (1 - 1/u_t)} - \beta_t \tag{31}$$

We may further observe that the term reflecting changes in the costs of operating the capital stock at other than normal capacity depends on the rate of capacity utilisation. But from equations (4) and (6) above, it is clear that this term could be expressed as a function of the rate of unemployment. As unemployment declines, output will be higher, and the rate of capacity utilization will also increase. This component of inflation will thus be inversely related to movements in the rate of unemployment. We could, therefore, approximate this factor with an additional term, $-\vartheta_3 UE_t$, that varies negatively with the rate of unemployment.

Replacing the additional capital cost term with this simplification and re-parameterizing so that $h_1 = \vartheta_1$, $\alpha = \vartheta_1 \cdot \psi$, and $h_2 = \vartheta_1 \cdot \vartheta_2 + \vartheta_3$, we obtain equation (24) in the model above. This equation does a similar job to the Rowthorn-Setterfield-Rochon model of conflict inflation. Workers formulate a target money wage increase each period which they hope to achieve but these aspirations are modified by the balance of industrial and political power. Whatever the money wage increase produced by this process, firms then raise prices to protect their current rate of profit. This process, therefore, has no actual effect on income distribution but only affects the rate of inflation. Distribution is determined purely by equation (10). The remaining equation (25) then determines inflation expectations via a standard adaptive process.

The structural form of our one sector Sraffa-Keynes model is thus comprised of 15 equations, (1)–(7), (9)–(14) and (24)–(25), in 15 unknowns, Y_t, C_t, I_t, K_t, u_t, ℓ_t, UE_t, $r^a_{n,t}$, $r^*_{n,t}$, w_t, $i_{WL,t}$, W_t, P_t, π_t and π^e_t. The following section obtains the reduced form for this structural model, finds the analytical solution to that reduced form, and explores the stability properties of the model's equilibrium values.

6. Reduced Form, Equilibrium and Stability

To obtain the model's reduced form, we begin with equation (1) which determines the system's output. Substituting (2) and (3) into this equation yields the following expression:

$$Y_t = \bar{C} + (1 - s_w) \cdot (1 - \tau) \cdot W_{t-1} + (1 - s_c) \cdot (1 - \tau) \cdot P_{t-1} + \delta \cdot \nu \cdot Y_{t-1} \cdot u_{t-1} + \bar{G} + \overline{NX} \qquad (32)$$

Gathering exogenous variables, lagging equations (13) and (14), and substituting for the aggregate wage and profit terms, W_{t-1} and P_{t-1}, gives the following expression after rearranging:

$$Y_t - \{w_{t-1} \cdot \ell_t \cdot (1 - \tau) \cdot (s_c - s_w) + (1 - s_c) \cdot (1 - \tau) \cdot (1 - \delta \cdot \nu \cdot u_{t-1}) + \delta \cdot \nu \cdot u_{t-1}\} \cdot Y_{t-1} = \bar{A} \qquad (33)$$

where $\bar{A} = \bar{C} + \bar{I} + \bar{G} + \overline{NX}$. Substituting (12) into (11) and assuming, according to the pseudo-Wray approach, that the central bank maintains a constant target for the nominal policy rate, gives:

$$r_{n,t} = i^* + \sigma_k \tag{34}$$

Then substituting this into the wage- profit relation in equation (10) gives:

$$w_t = \frac{1}{\ell_t} \cdot [1 - \nu \cdot (i^* + \sigma_k + \delta) - z_t \cdot (1 - 1/u_t)] \tag{35}$$

Substituting (35) for w_t into (33), and rearranging gives:

$$Y_t - \theta_1 \cdot Y_{t-1} = \bar{A} \tag{36}$$

where: $\lambda_1 = i^* + \sigma_k + \delta$;

$\lambda_2 = \left[1 - \nu \cdot \lambda_1 - z \cdot \left(1 - \frac{1}{u_t}\right)\right] \cdot (1 - \tau) \cdot (s_c - s_w)$;

$\lambda_3 = (1 - s_c) \cdot (1 - \tau) \cdot (1 - \delta \cdot \nu \cdot u_{t-1})$; and

$\theta_1 = \lambda_2 + \lambda_3 + \delta \cdot \nu \cdot u_{t-1}$.

Equation (36) is a second order non-linear difference equation in Y and u. We may obtain a second reduced-form equation in these variables by lagging equation (3) and substituting the resulting expression into equation (5) for the capital stock. This gives:

$$K_t = K_{t-1} \cdot (1 - \delta \cdot u_{t-1}) + \delta \cdot \nu \cdot Y_{t-2} \cdot u_{t-2} \tag{37}$$

From equation (4) we have:

$$K_t = \nu \cdot Y_t / u_t \tag{38}$$

Lagging this and substituting into (37) gives the following after some rearranging:

$$\delta \nu Y_{t-1} + \nu Y_t u_t^{-1} - \nu Y_{t-1} u_{t-1}^{-1} - \delta \nu Y_{t-2} u_{t-2} = 0 \tag{39}$$

Unemployment is given by equation (6), but if we assume that $\beta = 0$, so that $\ell_t = \ell_{t-1} = \ell$ from equation (5), we may express unemployment as follows, after minor rearranging:

$$UE_t = 1 - (\ell / \bar{N}) \cdot Y_t \tag{40}$$

Inflation may be expressed by lagging equation (24), substituting this into (25), and rearranging to obtain:

$$\pi_t^e - [1 - h_3(1 - h_1)] \cdot \pi_{t-1}^e = h_3 \cdot (\alpha - h_2 \cdot UE_{t-2}) \tag{41}$$

Equation (18) already expresses the actual rate of profit from equation (9) in terms of the target rate of profit, the rate of capacity utilization and the cost of running capital at other than full capacity. If we substitute (12) into (11) again assuming a constant policy rate, and we assume that the real cost of running the capital stock at anything other than full capacity is constant, equation (18) may be re-expressed as:

$$r_{n,t}^{a} = u_t \cdot (i^* + \sigma_k) - \frac{z}{v} \cdot (u_t - 1) \tag{42}$$

The model's reduced form is thus made up of five equations (36), (39), (40), (41) and (42) which are reproduced below for convenience:

$$Y_{t+1} - \theta_1 \cdot Y_t = \bar{A} \tag{36}$$

$$\delta v Y_{t+1} + v Y_{t+2} u_{t+2}^{-1} - v Y_{t+1} u_{t+1}^{-1} - \delta v Y_t u_t = 0 \tag{39}$$

$$UE_t = (\bar{N} - \ell \cdot Y_t)/\bar{N} \tag{40}$$

$$\pi_{t+1}^{e} - [1 - h_3(1 - h_1)] \cdot \pi_t^{e} = h_3 \cdot (\alpha - h_2 \cdot UE_{t-1}) \tag{41}$$

$$r_{n,t}^{a} = u_t \cdot (i^* + \sigma_k) - \frac{z}{v} \cdot (u_t - 1) \tag{42}$$

where: $\lambda_1 = i^* + \sigma_k + \delta$;

$\lambda_2 = \left[1 - v \cdot \lambda_1 - z \cdot \left(1 - \frac{1}{u_t}\right)\right] \cdot (1 - \tau) \cdot (s_c - s_w)$;

$\lambda_3 = (1 - s_c) \cdot (1 - \tau) \cdot (1 - \delta \cdot v \cdot u_{t-1})$; and

$\theta_1 = \lambda_2 + \lambda_3 + \delta \cdot v \cdot u_{t-1}$.

This is a system of second order non-linear difference equations in the variables, Y_t, u_t, UE_t, π_t^e and r_t^a that could be solved using appropriate non-linear techniques. An alternative approach, however, would be to linearize equation (36) by assuming that $u_{t-1} = u_t = 1$ which we expect to be true at equilibrium. Equation (36) would then become:

$$Y_t - \theta_1 \cdot Y_{t-1} = \bar{A} \tag{43}$$

where: $\lambda_1 = i^* + \sigma_k + \delta$;

$\lambda_2 = [1 - v \cdot \lambda_1] \cdot (1 - \tau) \cdot (s_c - s_w)$;

$\lambda_3 = (1 - s_c) \cdot (1 - \tau) \cdot (1 - \delta \cdot v)$; and

$\theta_1 = \lambda_2 + \lambda_3 + \delta \cdot v$.

Equations (39) and (42) would also become:

$$\nu Y_{t+2} - \nu(1-\delta)Y_{t+1} - \delta\nu Y_t = 0 \quad (44)$$

$$r^a_{n,t} = i^* + \sigma_k \quad (45)$$

Equations (43) and (44) are now second order linear difference equations in Y alone and are block recursive. The solution to this part of the reduced form model (shown in the Appendix) is given by:

$$Y^* = \frac{1}{1-\theta_1} \cdot \bar{A} \quad (46)$$

This solution is characterised by the standard Keynesian result that equilibrium output is the product of autonomous expenditures and a multiplier term. But here, the multiplier is a complex term that incorporates income distribution. Because consumption depends on the split of income between workers and capitalists, and monetary policy determines the rate of profit through the price setting behaviour of firms, equilibrium output depends on distribution as expressed by the central bank-determined policy rate. Equation (46) thus includes the present model's version of Serrano's (1995) supermultiplier. This also implies that money is *non-neutral* since the key nominal variable (the observed policy rate) affects the model's most important real variable. We also show in the Appendix that this solution is stable provided:

$$\theta_1 < 1 \quad (47)$$

We are also tempted to conclude that our full model is likely to be stable when this condition is satisfied. But since the assumption that $u_{t-1} = u_t = 1$ can be expected to hold only at equilibrium, and even a small disturbance of the full model away from equilibrium would violate this assumption, our approach will be to use equation (46) to derive the system's equilibrium, to simulate a number of shocks to that equilibrium, and then verify that the system converges to a stable long run time path characterised by the condition that $u_{t-1} = u_t = 1$.

According to equation (45) the actual rate of profit is equal to the target rate of profit when $u_{t-1} = u_t = 1$, and this is simply determined by monetary policy and the risk premium of capitalists. It will clearly be stable.

The equilibrium value for unemployment depends on the equilibrium value for output, Y^*. Once the latter is obtained from equation (46), this may be substituted into equation (40) to obtain:

$$UE^* = (\bar{N} - \ell \cdot Y^*)/\bar{N} \quad (48)$$

And the stability properties of this equilibrium depend directly on those of Y^*. The equilibrium value for expected inflation, π_t^e, is shown in the Appendix to be:

$$\pi_t^{e^*} = \frac{\alpha - h_2 \cdot UE^*}{1 - h_1} \tag{49}$$

which is stable if:

$$h_1 < 1 \tag{50}$$

This condition requires the coefficient on inflation expectations in equation (24) of the structural model to be less than unity. The value of this coefficient has been a significant point of difference between Keynesian economists, who assert that the long run Phillips curve is downward sloping, and monetarist or rational expectations economists, who assert that the long run Phillips curve is vertical (Gordon 2011, p.18). We thus assume that the long run Phillips curve is not vertical and that there is a trade-off between lower unemployment and higher inflation in the long run. Once again we will check that this condition is met when model calibration is considered in Section 7 below.

The logic of the reduced form system represented by equations (40), (41), (43), (44) and (45) is that monetary policy determines income distribution by setting the nominal policy rate which affects firms' choices of target return and price setting, and then the real wage via the wage-profit frontier. This then influences the level of output via the multiplier in equation (46). Once output is determined, the level of unemployment follows from equation (48), and this affects the bargaining power of workers in their negotiations with firms over nominal wage increases. This process determines inflation through the Phillips relation in equation (49). The system is, therefore, determinate and stable under reasonable conditions.

Before turning to the issue of model calibration, we reflect again on the relationship between the model's short period and long period characteristics. This can be understood in terms of equation (46), the expression for output equilibrium. Taking time derivatives of each side, dividing each side by output and multiplying by unity gives the following:

$$\frac{dY^*}{dt} \cdot \frac{1}{Y^*} = \frac{1}{1-\theta_1} \cdot \left(\frac{d\bar{A}}{dt} \cdot \frac{1}{\bar{A}} \right) \cdot \frac{\bar{A}}{Y^*} \tag{51}$$

The left hand side of (51) is the rate of output growth which is determined in this expression by the rate of growth in autonomous expenditures (given in the brackets on the right hand side) adjusted by the multiplier expression, $1/(1-\theta_1)$, and the proportion of autonomous expenditures in equilibrium income ($\bar{A}/Y^*$). Substituting for Y^*on the rate hand side of (51), it may be shown that the rate of growth of output and the rate of growth of autonomous expenditures must be equal. Dividing both sides by the rate of growth and rearranging gives the following:

$$\frac{\bar{A}}{Y^*} = \quad 1 - \theta_1 \tag{52}$$

Equation (52) is the present model's analogue of the Cambridge equation. Recall that θ_1 depends on the rate of profit (which in turn depends on monetary policy and σ_k, the profit "mark-up" applied by firms to the policy rate), the savings propensities of capitalists (s_c) and workers (s_w), the technical conditions of production, and the tax rate. Equation (52) thus indicates that long period equilibrium must be characterised by a relation between distribution, saving and the proportion of autonomous expenditures in total output. The growth rate does not feature in this relation since autonomous expenditures are not synonymous with investment spending as they are in the traditional treatment of the Cambridge equation (see, for example, Pasinetti 1974, pp.103-112). Growth is clearly driven by changes in total autonomous spending in this model (consumption, government spending and net exports). If this rate is equal to the sum of labour supply growth and growth in labour productivity, unemployment will remain constant on the long period growth path. If not, unemployment (and inflation) will systematically change, and the precise course of the economy will depend on policy responses to these trends.

It is straightforward to show that if (52) does not hold, output growth will differ from growth in autonomous spending with implications for both unemployment and inflation through time. If, for example, $\bar{A}/Y^* > \; 1 - \theta_1$, output growth would exceed growth in autonomous expenditures. If this rate of growth also exceeded the sum of labour force growth and growth in labour productivity, unemployment would be falling through time and inflation would be rising. This would provoke a policy response in the form of higher interest rates or higher tax rates, and since equation (46) above implies that $\partial(1 - \theta_1)/\partial i^* > 0$ and $(1 - \theta_1)/\partial\tau \; > 0$, either response would raise $1 - \theta_1$ and push the economy closer the equality expressed in equation (52).

As explained earlier in the paper, we assume that the secular growth rate in autonomous spending is zero in order to explore the short period features of the model. Variations in the model's variables considered below may then be interpreted as being *relative* to the economy's long run growth path. Since long period growth depends upon growth in autonomous spending, this seems to be a reasonable first step in exploring the short period features of the model. But changes in policy variables, for example, may affect the model's "Cambridge equation" thus having long run implications. We comment on this issue in the conclusion but leave detailed analysis of this potential for interaction to future work.

7. Model Calibration

Table 1 sets out the values for parameters and exogenous variables that were chosen in order to simulate two kinds of macroeconomic shock for the model described in Sections 3 - 5 above. These values were calibrated against the broad features of the Australian economy. The objective was not, however, to precisely replicate these features but to create a model economy with plausible dimensionality, and features of the Australian economy were used as a guide for the construction of that dimensionality. One period in our discrete time set up within the Sraffa-Keynes model was, therefore, interpreted as being one quarter (three months) in terms of real time.

The level of output was initially set at a value of 2000 but adjustments to this value were allowed in order to accommodate the dimensionality requirements of other variables. The savings propensity for workers was chosen, for example, to reflect the general saving characteristics of Australian households. The net savings propensity of the Australian household sector between 1960 and 2019 is shown in Panel A of Figure 1. A downward trend is observable in this data commencing in about 1981 from the 1960-1980 average of about 14% to 0% in 1999-2000. The average since 2000 has been about 3.9%. We base the savings propensity of workers in the Sraffa-Keynes model on this post 2000 average, setting the value at 0.04. We then set the savings propensity of capitalists at 0.20. The latter is somewhat arbitrary in the absence of data about the savings habits of higher income Australian households but this value is a little above the 1960-1980 average of 14% to reflect the habits of a more parsimonious age and the likelihood that higher income households would have saved above this average value across all households in that period.

The tax rate of 0.26 in Table 1 was calculated by taking the total tax receipts of the Australian Commonwealth Government for the period 1 July 2017 to 30 June 2018 and dividing this by nominal GDP for the same period.[9] These tax receipts included household income tax, business income tax, and indirect taxes rather than simply income tax as modelled

[9] The nominal value of these tax receipts was $AUD 416,352 million (Commonwealth of Australia 2018, p.5-5). GDP at current prices from Sept Q 2017 to June Q 2018 was $AUD 1,848,062 million (Australian Bureau of Statistics, Catalogue No. 5206.0, *Australian National Accounts: National Income, Expenditure and Product*, Table 3 – Expenditure on Gross Domestic Product, Current Prices). Dividing this measure of tax receipts by GDP gave an average tax rate of 23.5%.

Table 1: Parameter and Exogenous Variable Calibration

Variable	Description	Initial Value
Parameters		
s_w	Savings propensity of workers	0.04
s_c	Savings propensity of capitalists	0.20
τ	Tax rate	0.26
δ	Depreciation rate	0.03
v	Capital-output ratio	4.50
ℓ	Labour-output ratio	6.96
h_1	Sensitivity of inflation to expected inflation	0.70
h_2	Sensitivity of inflation to rate of unemployment	0.28
h_3	Expected inflation adjustment parameter	0.40
Exogenous variables		
$\bar{C}$	Autonomous consumption	40
$\bar{G}$	Autonomous government spending	480
$\overline{NX}$	Autonomous net exports	25
$\bar{N}$	Labour supply	13,800
α	Base per cent increase in money wages	0.024
β	Per cent per period increase in labour productivity	0
σ_k	Nominal premium expected on capital over wholesale rate	0.05
$i*$	Nominal wholesale rate set by central bank	0.03

in the Sraffa-Keynes framework. But since the tax rate, τ, represents the only form of taxation in the model, it seemed reasonable to calibrate this to total taxation for the Australian economy with a small upwards revision. The average rate of tax obtained by this method was 23.5%. An alternative method would be to divide only income tax receipts for the same period by the sum of wage income and profit income for that period.[10] This method produced an average tax rate of 23.1%. These figures were increased slightly to 26% or 0.26 again to accommodate the need for the values of other variables such as the ratio of consumption to GDP or the size of the budget deficit relative to GDP to be plausible.

Expression (14) from above may be employed to calibrate the depreciation rate and the capital-output ratio:

[10] "Income taxation receipts" for the 2017-2018 year were $AUD 300,670 million according to the Australian Treasury's *Mid-Year Economic and Fiscal Outlook* (Commonwealth of Australia 2018, p.5-15). Total incomes were calculated as the sum of "Total compensation of employees", "Gross operating surplus – total corporations" and "Gross mixed income" for the period Sept Q 2017 to June Q 2018 from Australian Bureau of Statistics, Catalogue No. 5206.0, *Australian National Accounts: National Income, Expenditure and Product*, Table 7 – Income from Gross Domestic Product, Current Prices. This figure was $AUD 1,360,439. Dividing this measure of tax receipts by the measure for total incomes gave an average tax rate of 23.1%.

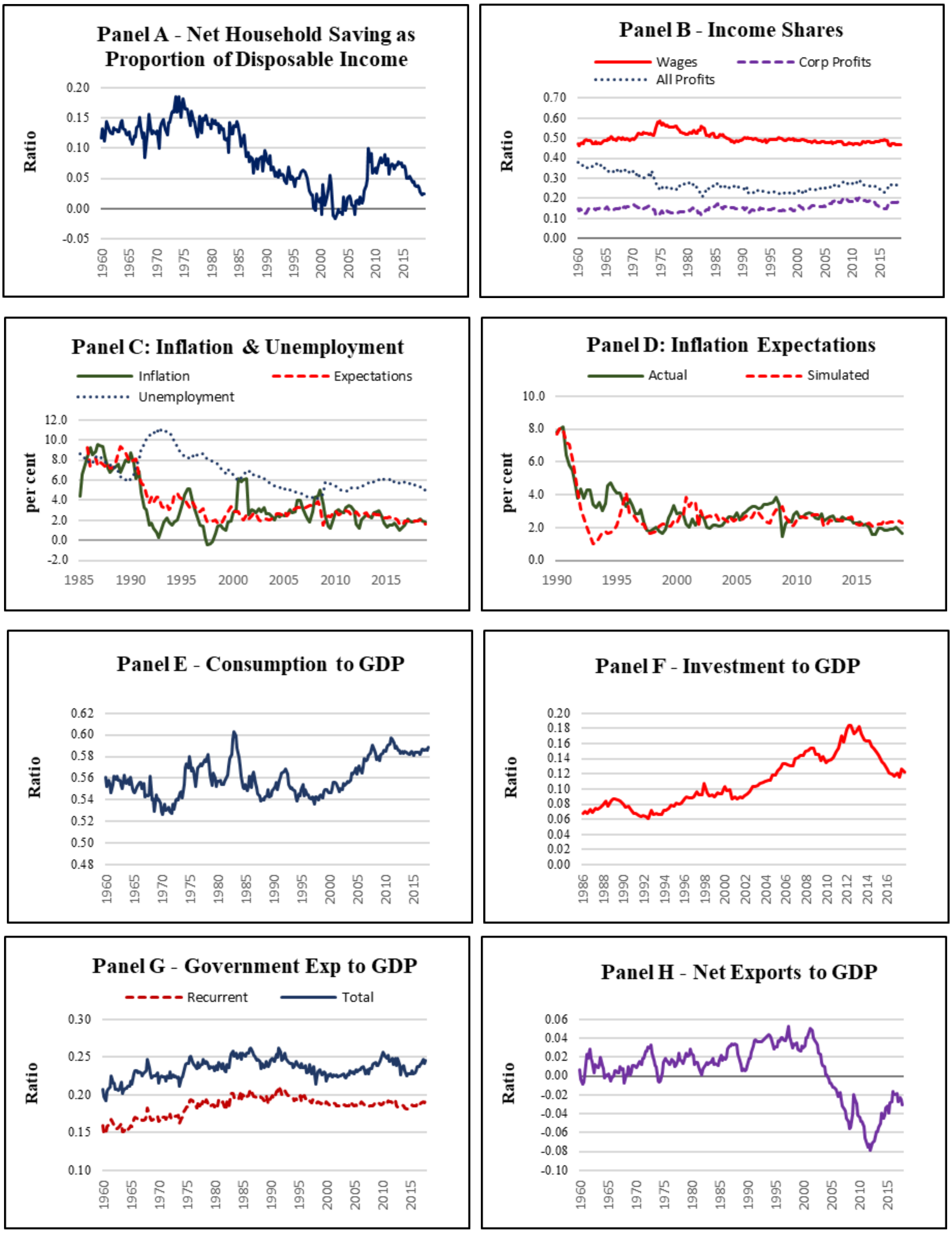

Figure 1: Savings, Expenditure and Income Shares for the Australian Economy

Source: Australian Bureau of Statistics, Catalogue No. 5206.0, *Australian National Accounts: National Income, Expenditure and Product*, Table 20: Household Income Account, Current prices, seasonally adjusted; Table 7: Income from Gross Domestic Product, Current prices, seasonally adjusted; and Table 3: Expenditure on Gross Domestic Product (GDP), Current prices, seasonally adjusted; Catalogue No. 6202.0, *Labour Force*, Table 1 - Labour Force Status by Sex, Australia; and Table 22 - Underutilized Persons by Age and Sex; Catalogue No. 6401.0, *Consumer Price Index, Australia*, Tables 1 and 2 - All Groups Index Numbers and Percentage Changes; Reserve Bank of Australia, *Table G3 Inflation Expectations*.

$$P_t = Y_t - W_t - \delta \cdot \nu \cdot Y_t \cdot u_t \tag{14}$$

If we assume that $u_t = 1$, divide this expression through by Y_t, and rearrange, we obtain:

$$W_t/Y_t + P_t/Y_t + \delta \cdot \nu = 1 \tag{53}$$

which indicates that the wage share of total output, the profit share, and depreciation per unit of output must sum to unity. With assumptions about the wage and profit shares, equation (51) can be used to determine the $\delta \cdot \nu$ term. Panel B of Figure 1 shows the wage and profit shares for the Australian economy from 1960 until 2019. The wage share is relatively stable for this period around a value of 0.50, and we, therefore, adopted this value. Two profit shares are shown in Figure 1, one that includes only profits of the corporate sector, and one that includes small business income (labelled "All Profits"). The former fluctuates between about 0.12 in the early 1960s to about 0.20 in 2019 (an average of 0.155 across the entire period). The latter falls from about 40% in the early 1960s to just below 30% in 2019 (an average of 0.273). We initially assumed a share of 0.36 to account for the absence of other income shares in the Australian context. But this seemed reasonable given the history of this variable. Substituting these assumptions into (51) above gives a value of 0.14 for $\delta \cdot \nu$. We then used the following definition of the rate of profit as the real value of profits divided by the real value of capital to infer the value of ν:

$$r = P_t/K_t$$

Noting that in our model $K_t = \nu \cdot Y_t$ allows us to write:

$$r = P_t/(\nu \cdot Y_t)$$

since P_t/Y_t is simply the profit share, which we have already assumed to be 0.36, we may infer the value of ν from this expression by making some assumption about the rate of profit. We assumed a rate of profit for this purpose of 8%. Thus: $\nu = 0.36/0.08 = 4.5$. Since $\delta \cdot \nu = 0.14$, a value of 4.5 for ν implies $\delta = 0.0311$, which we rounded to 0.03. We adopted these values as shown in Table 1.

The Australian labour force at 30 June 2018 was 13,287,700 in seasonally adjusted terms.[11] This was rounded up slightly to 13,800,000. The labour output ratio ℓ was initially set by dividing this labour force size by the initial output value of 2,000, giving a value of $\ell = 6.9$.

[11] Australian Bureau of Statistics, Catalogue No. 6202.0, *Labour Force, Australia*, Table 1: Labour Force Status by Sex, Australia.

This value was subsequently adjusted slightly to ensure that other dimensionality settings could be accommodated since the precise meaning of this term in a one sector model has no direct analogue in the Australian economy. The final value chosen was 6.96.

The values for h_1 and h_2 in the Phillips equation (24) were chosen using the naive approach of estimating simple OLS regressions of this equation on Australian data. While such an approach ignores a number of econometric issues, it was judged that the estimates obtained would provide values of sufficient proximity to the characteristics of the Australian economy for our purposes. Panel C of Figure 1 plots CPI inflation, financial market inflation expectations, and the unemployment rate for the period 1985 to 2018 in Australia. We ran OLS regressions of the Phillips curve relation for the 1993-2018 sub-period, which corresponds to the Reserve Bank of Australia's adoption of inflation targeting. The specification used is shown in equation (54) below which contains a dummy variable for the introduction of the *Goods and Services Tax* (GST) in 2000 since this clearly caused a one-off jump in the inflation rate that year. The equation was estimated using both unemployment and under-employment data, and using two measures of inflation expectations, that implied by the difference in financial markets between the interest rate on 10 year Government bonds and the rate on 10 year inflation-indexed Government bonds, and that derived from a survey of business leaders. The $\mu's$ are all coefficients, and ϵ_t is a random error term.

$$\pi_t = \mu_0 + \mu_1 \cdot \pi_t^e + \mu_2 \cdot UE_{t-1} + \mu_3 \cdot GST + \epsilon_t \tag{54}$$

The results of these regressions are shown in Table 2. The coefficient on inflation expectations has the right sign and is noticeably less than unity for all variants of the models estimated. We showed in Section 6 that the equilibrium values for inflation and expected inflation in the Sraffa-Keynes model were stable provided this coefficient value was less than unity. There is, however, a noticeable difference in the estimated coefficients for this variable when financial markets expectations were used compared to survey-based expectations. Given the higher *t*-statistics for the financial market estimates, and the higher *F*-statistics and R^2s for the regressions using financial market expectations, it was decided to use estimates from these regressions in the model. An approximate average was taken of the estimate of this parameter from the regressions using unemployment and under-employment respectively, to arrive at a parameter value for h_1 of 0.7 which is the value shown in Table 1. The coefficient on unemployment also has the expected negative sign but is more consistent across variants of the models estimated. We thus chose a value for h_2 of 0.28 which is also shown in Table 1.

Table 2: Simple OLS Estimates of Australian Phillips Curve Parameters

Parameter	Unemployment		Under-employment	
	Financial Market Expectations	Business Expectations	Financial Market Expectations	Business Expectations
Constant	2.1664 (5.2669)	3.3682 (8.6008)	4.5140 (6.2133)	5.5999 (7.5166)
Expectations	0.8166 (6.1161)	0.3995 (2.7609)	0.6089 (4.8446)	0.2515 (1.9340)
Unemployment	-0.3237 (-5.8517)	-0.2704 (-4.2504)	-0.2864 (-5.7013)	-0.2758 (-5.0013)
GST	4.0100 (8.9176)	3.7443 (7.4079)	3.7920 (8.3935)	3.6063 (7.3420)
Adjusted R^2	0.5319	0.4024	0.5258	0.4356
F-Statistic	40.0205 (0.0000)	24.1206 (0.0000)	39.0711 (0.0000)	27.5013 (0.0000)

Notes: Regressions were estimated with Australian data for the period 1993-2018. Numbers in parentheses are t-statistics except for F-statistics where they indicate p values. *Data Sources*: Australian Bureau of Statistics, Catalogue No. 6202.0, *Labour Force*, Table 1 - Labour Force Status by Sex, Australia; and Table 22 - Underutilized Persons by Age and Sex; Catalogue No. 6401.0, *Consumer Price Index, Australia,* Tables 1 and 2 - All Groups Index Numbers and Percentage Changes; Reserve Bank of Australia, *Table G3 Inflation Expectations*.

The inflation expectations adjustment parameter (h_3) in equation (25) was initially set at a mid-range value of 0.5. Expected inflation was then simulated for the period 1990 to 2018 using this value, and the simulated values were compared to actual values. We then experimented with small variations in the parameter value (from 0.4 to 0.6) and found that a value of 0.4 generated the smallest mean squared error relative to actual values. Panel D in Figure 1 shows simulated expectations using this value against actual inflation expectations. The fit indicated by this figure was judged adequate for use in our simulations.

This accounts for the calibration of each parameter in the upper section of Table 1. Values for the exogenous variables in the lower section of Table 1 were chosen so as to generate values for endogenous variables and key macroeconomic ratios such as wage and profit shares, consumption to GDP, investment to GDP, government spending to GDP, net exports to GDP, and the fiscal surplus to GDP ratio that were close to actual values for the Australian economy shown in Panels E to H of Figure 1. Once again, the objective was not to replicate such values but to generate a hypothetical economy with plausible dimensionality. The resulting values for endogenous and indicative macroeconomic variables are shown in Table 3. These were used as the initial values for the demand and cost shock simulations.

Table 3: Initial Equilibrium Values for Variables in the Model

Variable	Variable Name	Initial Equilibrium Value
Endogenous Variables		
Y	Output (real)	1,859.26
C	Total consumption spending (real)	1,103.26
I	Investment spending (real)	251.00
UE	Unemployment rate	0.0623
π	Inflation rate♦	0.0219
π^e	Expected inflation rate♦	0.0219
W	Total wages (real)	938.92
P	Total profits (real)	669.33
w	Real wages rate (per worker)	0.0726
r_n	Rate of profit (nominal)†	0.0800
Other indicative variables		
	Total employment	12,940.42
	Depreciation (real)	251.00
	Capital Stock (real)	8,366.65
	Wage share	0.5050
	Profit share	0.3600
	Depreciation share	0.1350
	Consumption spending as proportion of output	0.5934
	Investment spending as proportion of output	0.1350
	Government spending as proportion of output	0.2582
	Net Exports % output	0.0134
	Total tax receipts (real)	418.15
	Budget surplus (% output)	– 3.33

♦ Note that inflation rates are interpreted as annual rates over the previous four quarters (periods) but are calculated on a rolling quarter by quarter (period by period) basis. †We assumed that *n* in equation (11) was equal to 5, so that firms look back over the previous 5 quarters to determine the average policy rate on which to apply the profit "mark up" that determines the target rate of profit.

The remaining calibration task was to determine an appropriate size for simulation shocks. For the aggregate demand shock, it was decided to reduce the value of autonomous consumption, the question was by how much and for what duration? Figure 2 provides some perspective from the two most recent Australian recessions prior to the pandemic of 2020. Panel A shows annual GDP growth between 1960 and 2020, and Panel B shows this growth on a quarterly basis. The recessions of 1982-83 and 1991 stand out clearly in these panels. Panels C and D focus on these two recessions and the quarters around them, showing annual growth rates by quarter. Panel C indicates that GDP growth was negative or close to zero from the third quarter of 1982 until the second quarter of 1983 (inclusive), while panel B indicates negative

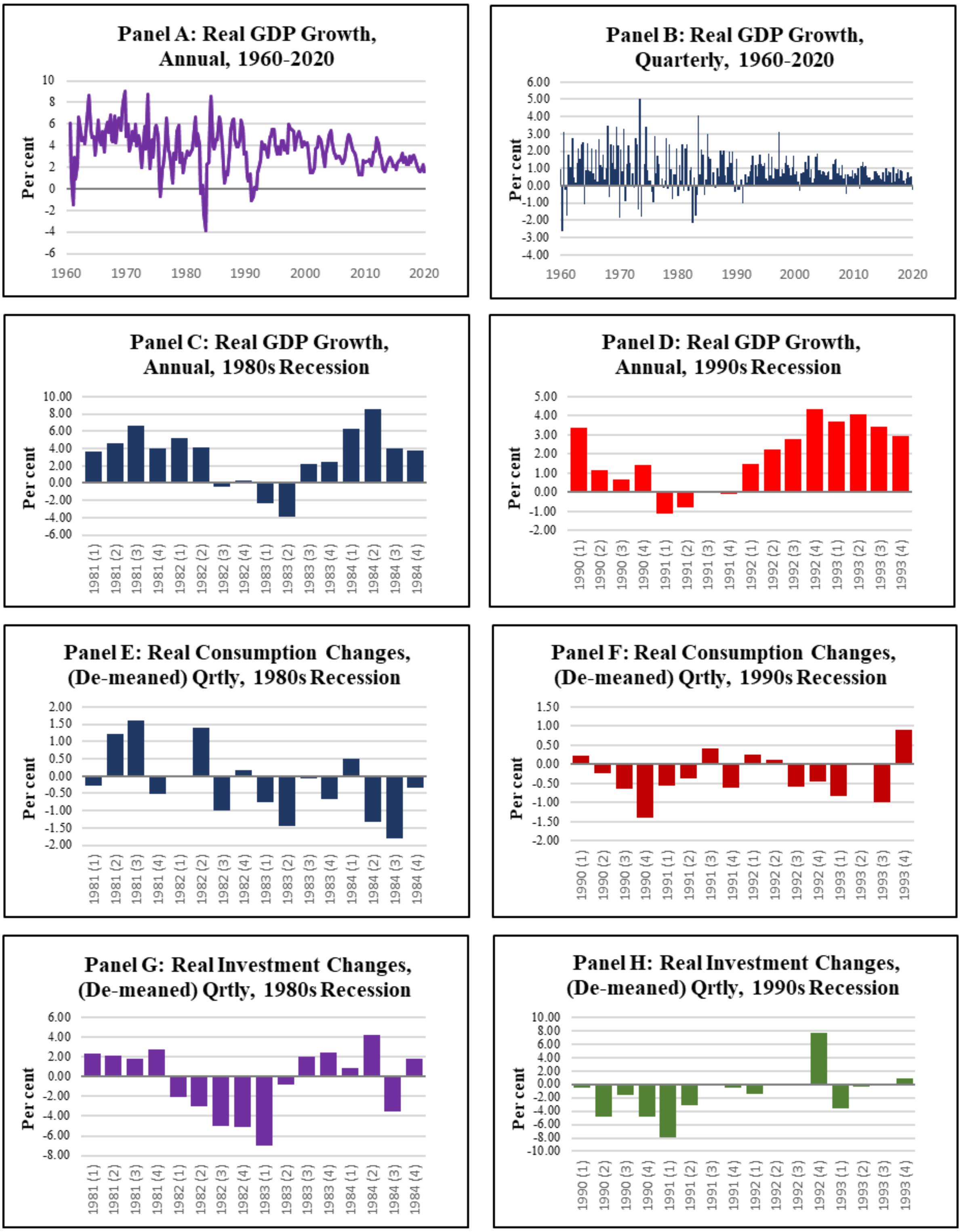

Figure 2: Australian GDP Growth, 1960-2020 and Quarterly Demeaned Changes in Consumption and Investment Spending during 1980s and 1990s Recessions.

Source: Australian Bureau of Statistics, Catalogue No. 5206.0, *Australian National Accounts: National Income, Expenditure and Product*, Table 2: Expenditure on Gross Domestic Product (GDP), Chain Volume Measures, seasonally adjusted.

annual growth for the first and second quarters of 1991 and close to zero growth for the third and fourth quarters of that year. The first of these recessions can thus be identified as occurring from 1982(3) to 1983(2) inclusive, and the second from 1991(1) to 1991(4) inclusive.

Panels E to H show de-meaned quarterly growth in two key components of aggregate demand, consumption and investment spending, for the same time periods as Panels C and D. Mean quarterly growth rates for these components were calculated for sub-periods around the two recessions and subtracted from their actual quarterly growth rates in order to provide an indication of variations relative to their long run growth paths. This was done to proxy the conceptual set up in the model outlined above where we assume zero growth. Panels G and H show a clear pattern of negative variations in investment spending leading up to the two recessions. De-meaned investment spending declines by about 2% and 3% respectively in the two quarters before the onset of recession in the third quarter of 1982, but falls for a further four quarters (the duration of the recession) after this. There are also negative changes to investment in the four quarters prior to the 1991 recession amounting to a cumulative reduction of about 10%. Panels E and F indicate that quarterly changes in de-meaned consumption spending is more volatile around these recessions although for the three quarters prior to the onset of the 1991 recession, there is a persistent fall in consumption amounting to about 2.25%.

Table 4 expresses these quarterly changes in consumption and investment as changes in overall aggregate demand. Percentage changes in consumption and investment are shown for the two quarters immediately before the onset of the two recessions, and for these two quarters plus the first two quarters of the recessions themselves. The shares of consumption and investment in GDP at the onset of these recessions (as indicated in Figure 1) are also shown.[12] If the percentage changes in each component of aggregate demand is weighted by its share in GDP, an estimate of the contribution of these changes to total aggregate demand is obtained, and these contributions are also shown in Table 4.

We thus see that the total change in aggregate demand from these two sources (that make up approximately 75% of aggregate demand in Australia) for the two quarters prior to the onset of the 1982-83 recession was a small increase of 0.60% and for the four quarters around the onset of this recession was a decline of 0.18%. For the 1991 recession, these respective changes were a decline of 1.76% for the two quarters prior to the onset of the recession, and a decline of 3.04% for the four quarters around this onset. Were we to generate a downturn in aggregate

[12] The time series for investment in Figure 1 only begins in 1986. We, therefore, assumed that the proportion of investment spending in 1982 was the same as its value in 1986 for the purposes of Table 4.

Table 4: Key Components of Change in Aggregate Demand in Australian Recessions

Component of Aggregate Demand	2 Period Change*			4 Period Change*		
	Change in Component	Propn GDP	Change in AD	Change in Component	Propn GDP	Change in AD
1982-83 Recession						
Consumption	+ 1.50%	0.60	+ 0.90%	+1.00%	0.60	+ 0.60%
Investment	– 5.00%	0.06	– 0.30%	– 13.00%	0.06	– 0.78%
Total			+ 0.60%			– 0.18%
1991 Recession						
Consumption	– 2.00%	0.56	– 1.12%	– 3.00%	0.56	– 1.68%
Investment	– 8.00%	0.08	– 0.64%	– 17.0%	0.08	– 1.36%
Total			– 1.76%			– 3.04%

* *Note*: The *2 Period Change* refers to the total change in the respective component of aggregate demand in the two quarters prior to the onset of the respective recession (1982(3) for the 1982-83 recession and 1991(1) for the 1991 recession). The *4 Period Change* refers to the total change in the respective component of aggregate demand in the two quarters prior to the onset of the recession plus the first two quarters of the recession itself.

demand of this magnitude in the Sraffa-Keynes model, but to focus this shock solely on an initial reduction in autonomous consumption, the two period shock of −1.76% from the 1991 recession would translate into a 2.98% decrease in consumption, and the four period shock of −3.04% would translate into a 5.15% decrease in consumption.[13] These shocks imply unit changes in autonomous consumption of 32.91 units across two quarters and 56.85 units across four quarters.[14] Distributing these reductions in autonomous consumption evenly across these respective quarters gives us reductions in 16.46 units per quarter for the two quarter shock, and 14.21 units per quarter for the four quarter shock. Averaging these values gives us a quarterly reduction of 15 units in autonomous consumption. We decided to scale this up by 30% to generate a sizeable downturn in our simulated economy and to do this for two quarters. Autonomous consumption was thus reduced by 20 units for two quarters in period 51 for the negative demand shock.

The inflation shock was much more straightforward to determine. Here we simply increased

[13] Given that the share of consumption in GDP in the Sraffa-Keynes model is 0.59, a 1.76% decrease in aggregate demand requires a 1.76%/0.59 = 2.98% decrease in consumption. A 3.04% reduction in aggregate demand requires a 3.04%/0.59 = 5.15% reduction in consumption.

[14] Since the equilibrium value of consumption from Table 3 is 1,103.26 units, the associated two quarter decrease in the *amount* of autonomous consumption is 1,103.26 × 2.98% = 32.91 units and the associated four quarter decrease is 1,103.26 × 5.15% = 56.85 units.

the base demand for money wages, α in equation (24), upwards by 1% in period 51 for four quarters. This translates into an increase in inflation of 1%. This is a modest increase but with an equilibrium inflation rate of between 2% and 3% in Table 3, this would push inflation beyond the boundary at which the Reserve Bank of Australia would respond with tighter monetary policy, and was thus judged to constitute a reasonable experiment.

8. Simulation Results

In Section 6 above, we derived the model's equilibrium and explored its stability properties on the assumption that $u_{t-1} = u_t = 1$. We argued that this was a reasonable assumption since we expected this condition to hold at equilibrium but noted that it should be checked when the model was simulated. It is worth noting that failure of this assumption to hold particularly affects the investment function in equation (3) of the structural model, the realised or actual rate of profit in equation (9), the real wage in equation (10), and associated pricing decisions in equation (22). We must, therefore, verify that the system gravitates to a stable time path after it experiences some shock, and that this equilibrium time path is characterised by a rate of capacity utilization equal to 1 in order to confirm that results from Section 6 hold. We will do this using two types of shock to the initial equilibrium defined by the values for endogenous and indicative macroeconomic variables listed in Table 3: a negative demand shock; and a positive cost shock. We consider each shock in turn.

(a) Negative Demand Shock

We first consider a *permanent* version of the negative shock of 20 units to autonomous consumption spending described in the previous section in order to verify the long period features of the model. Autonomous consumption thus falls in period 51 and remains there. We do this for both the *constant nominal rate* monetary policy setting described earlier in the paper, where the nominal policy rate is set at a particular level and left, and for the *nominal* e*mployment target* setting where the nominal rate is set at the rate consistent with a particular unemployment target and only recalibrated when there is a change to the structure of the economy. This is precisely what happens when autonomous consumption falls in period 51, and so this policy recalibrates the value of *i** set by the central bank (in equation (12) of the structural model to deliver an unemployment rate of 6.23%) when this occurs. The equation for finding this value of *i** is derived in the Appendix, and application of this equation to the negative demand shock implies that *i** falls from its initial value of 0.03 (3%) as indicated in Table 3, to 0.02 (2%). We also assume for this initial simulation that prices are maintained at their long period levels and are not adjusted across the economic cycle.

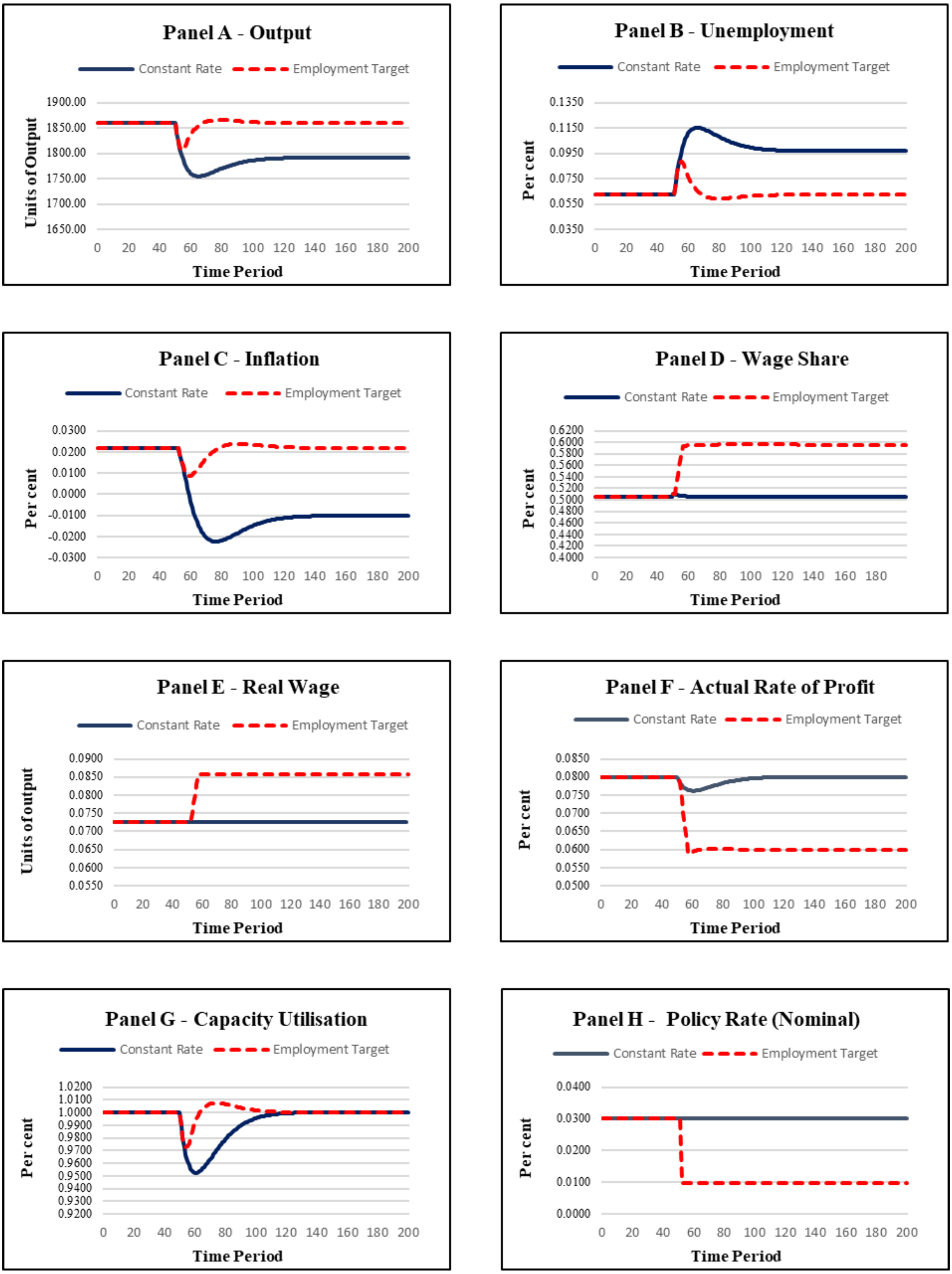

Figure 3: Time Paths of Key Variables for Negative Demand Shock (Permanent) – *Constant Nominal Rate* versus *Nominal Employment Target* for Monetary Policy (Constant Sraffa Prices)

The time paths for key variables, particularly capacity utilisation, the actual rate of profit and the real wage are shown as the solid lines in Figure 3 under the *constant nominal rate* approach to monetary policy. While Panels A, B and C indicate that output falls, unemployment rises and inflation falls (all permanently) as a result of this negative shock, capacity utilisation in Panel G initially falls, but eventually returns to its desired level of unity as a result of responses in investment spending to utilisation levels that are initially too low. As a result of this, the actual rate of profit, shown in Panel F, falls initially below its long period target as output declines, but eventually returns to this target as the capital stock is adjusted in response low levels of capacity utilisation and the capacity utilisation rate returns to unity. Since prices are not adjusted across the cycle in this case, there is no change to the real wage as indicated by the solid line in Panel E. The wage share does, however, rise temporarily since although employment decreases after the shock and this has a negative effect on the wage share for a given real wage, the actual profit rate declines as capacity utilisation falls, and this effect dominates the overall effect on income distribution.

The time paths for these variables under the *nominal employment target* rule are shown by the dashed lines. Panel H indicates a permanent adjustment to the policy rate in order to maintain equilibrium unemployment at its initial value of 6.23%. Panel B indicates that this policy adjustment is effective with the unemployment rate returning to this value within approximately 50 quarters despite the persistent decline in autonomous consumption. Output and inflation show similar patterns of returning to their initial levels in Panels A and C. Capacity utilisation in Panel G similarly returns to its long run value after experiencing a lower degree of volatility given the policy rate adjustment.

There are of course permanent changes to income distribution as a result of the change to the policy rate. The target rate of return is permanently reduced in response to the lower policy rate (see Panel F) and this generates a higher real wage (see Panel E). This translates into a permanent redistribution from capital to labour as reflected in the persistent increase in the wage share shown in Panel D.

Overall, Figure 3 indicates that the Sraffa-Keynes model outlined above with investment responding to variations in capacity utilisation, is stable in the long run, and is characterised by capacity utilisation at its desired level of unity with the long period rate of profit at its target level. The conclusions drawn in Section 6 about equilibrium and stability are thus confirmed.

The model simulated in Figure 3 maintained however, prices at their normal or long period values. We next explore the effect of allowing these prices to vary across the cycle. This is done in Figure 4 where the model was simulated with a *constant nominal rate* for monetary

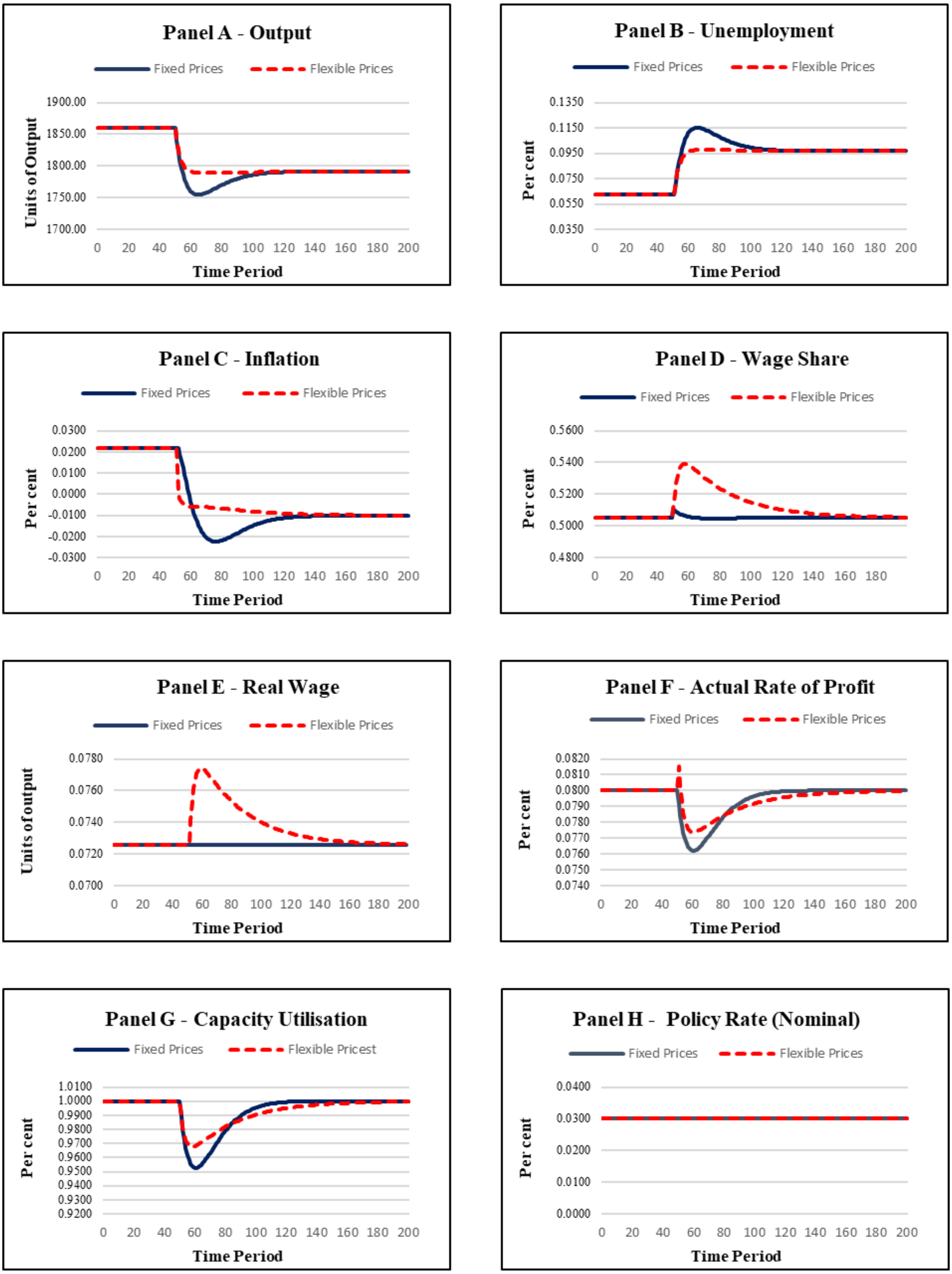

Figure 4: Time Paths of Key Variables for Negative Demand Shock (*Permanent*) and *Constant Nominal Rate* Approach to Monetary Policy - Fixed versus Flexible Prices

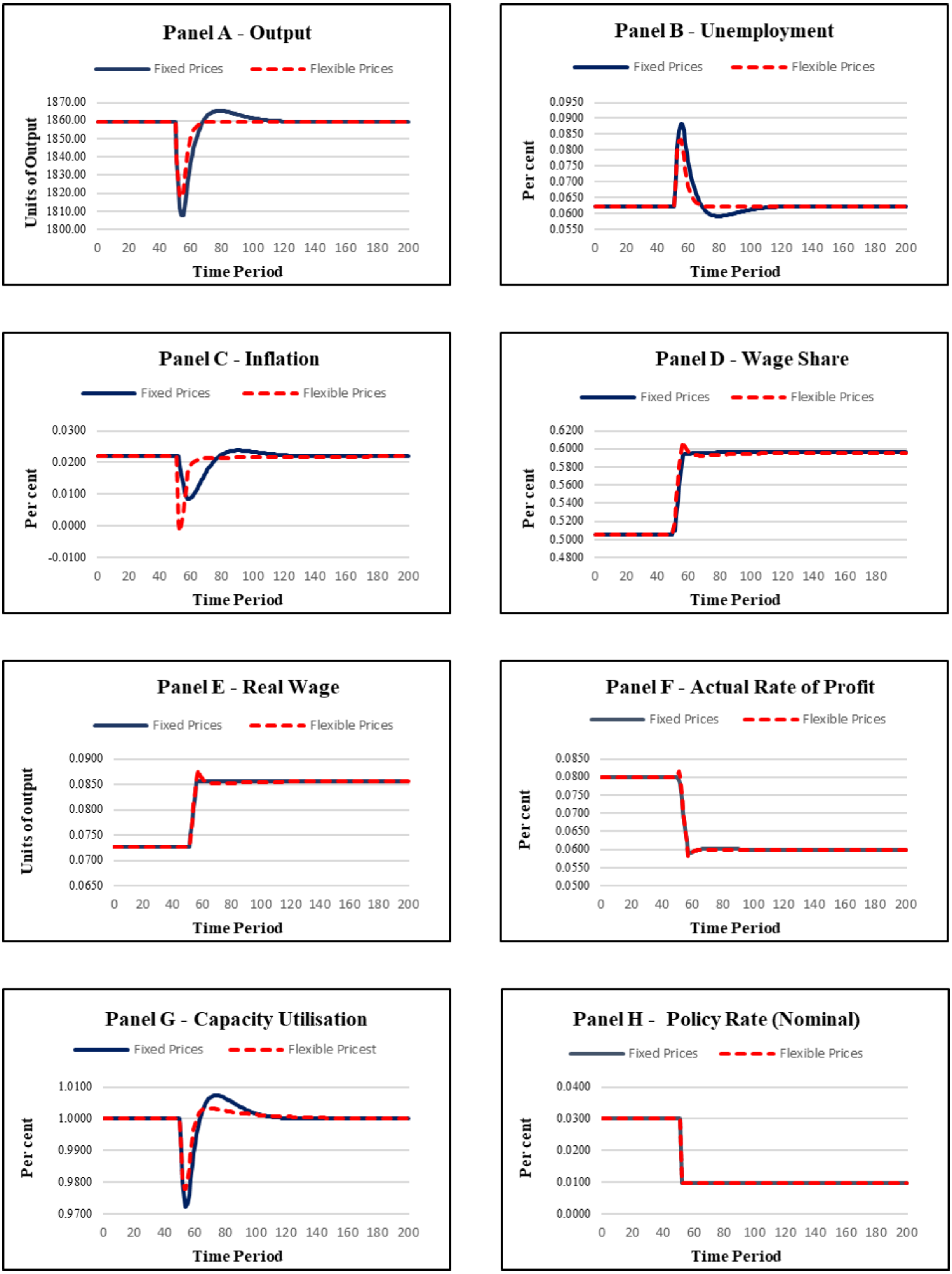

Figure 5: Time Paths of Key Variables for a Negative Demand Shock (*Permanent)* and a *Nominal Employment Target* Approach to Monetary Policy - Fixed versus Flexible Prices

policy and alternative approaches to pricing indicated in equation (22) depending on whether $z = 0$ or not. We refer to the case in which $z = 0$ as the *fixed price case* where firms maintain prices at their long period level corresponding to Sraffian prices of production even across the cycle. This case is shown by the solid lines in Figure 4. We refer to the case where $z \neq 0$ as the *flexible price case* in which firms adjust prices across the cycle in response to the additional costs imposed by running the capital stock at other than desired capacity. This is indicated by the dashed lines in Figure 4. Notice for this case that as the rate of capacity utilisation falls (see Panel G) after the negative shock hits in period 51, there is an adjustment of prices downwards according to equation (22), and this raises the real wage (assuming fixed money wages) as indicated in Panel E. A higher real wage raises the wage share (as indicated in Panel D), and this moderates the initial decline in aggregate demand and output as indicated in Panel A, compared to the fixed price case represented by the solid line.

For output, unemployment and inflation, flexible prices moderate volatility as they adjust to new long run equilibrium levels after the permanent decline to autonomous consumption in period 51. The volatility of capacity utilisation is also reduced in the flexible price case for the same reason, but as before, capacity utilisation returns to its desired level of unity in the long run. This further implies that all distributive variables eventually return to their long run values when prices are once again long period prices in the Sraffian sense.

Figure 5 reports simulation results for the permanent demand shock in the fixed and flexible price cases when monetary policy is governed by the *nominal employment target* rule rather than the simpler *constant nominal rate* approach. In this case, the effect of flexible prices is dominated by the impact of adjustments to the policy rate. While the effects of flexible as opposed to fixed prices remain, so that output, unemployment and inflation achieve their long run values sooner when prices are flexible rather than fixed, these differences are relatively small. Upon reflection, this result is not surprising since price flexibility moderates changes in the real economy via changes to income distribution, and this is precisely what adjustment to the policy rate does when its underlying equilibrium value is modified after the demand shock.

Having verified the long period characteristics of the full structural model, we may thus go on to consider the nature of a *transitory* demand shock. This shock will be similar to the permanent shock just considered but will only last for 2 quarters as described in the previous section. Autonomous consumption will thus return to its original value in period 53. The effects of this shock are shown in Figure 6 for the fixed and flexible price cases under the *constant nominal rate* approach to monetary policy. In this case, output, unemployment,

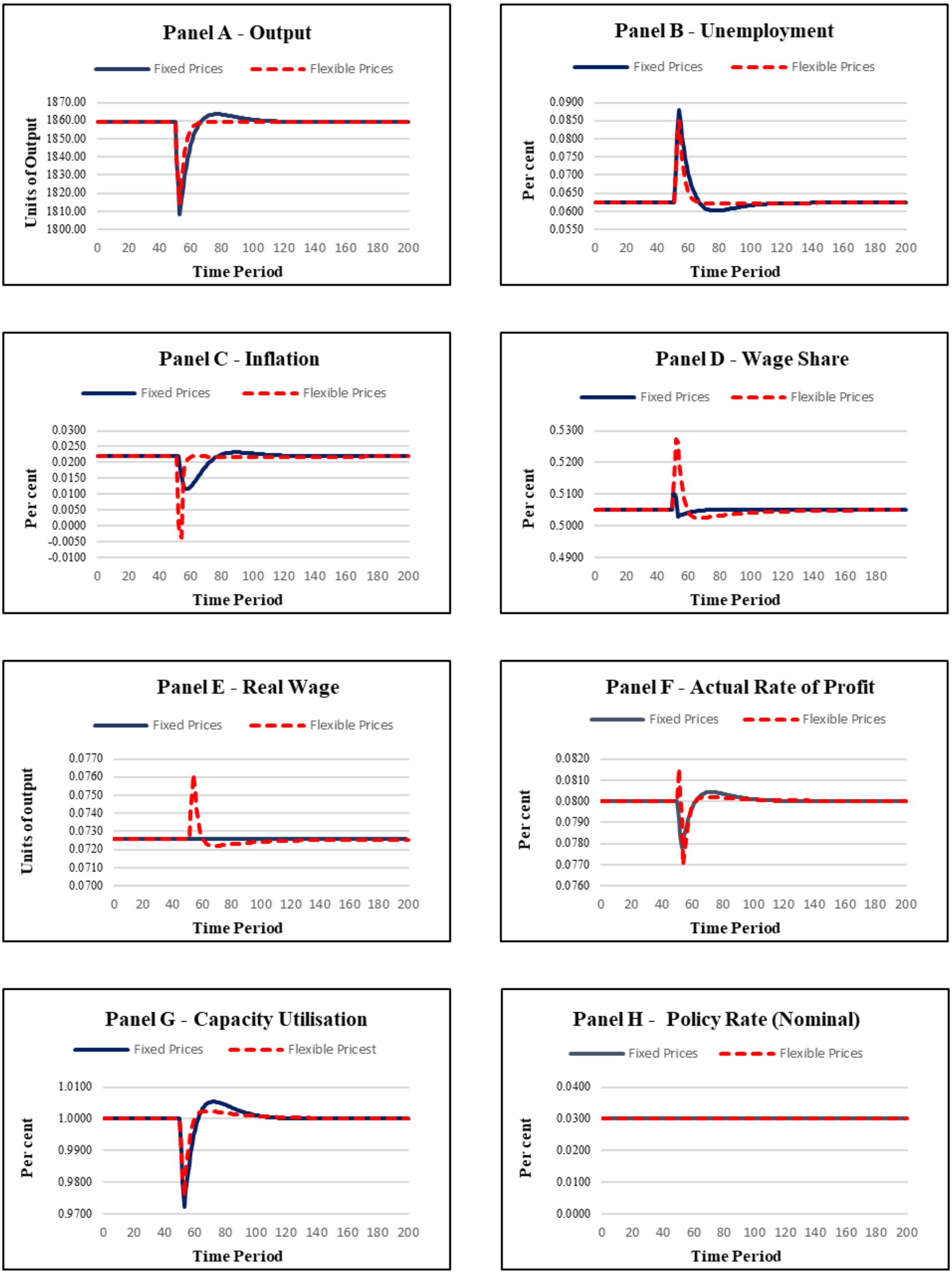

Figure 6: Time Paths of Key Variables for Negative Demand Shock (*Transitory*) and *Constant Nominal Rate* for Monetary Policy - Fixed versus Flexible Prices.

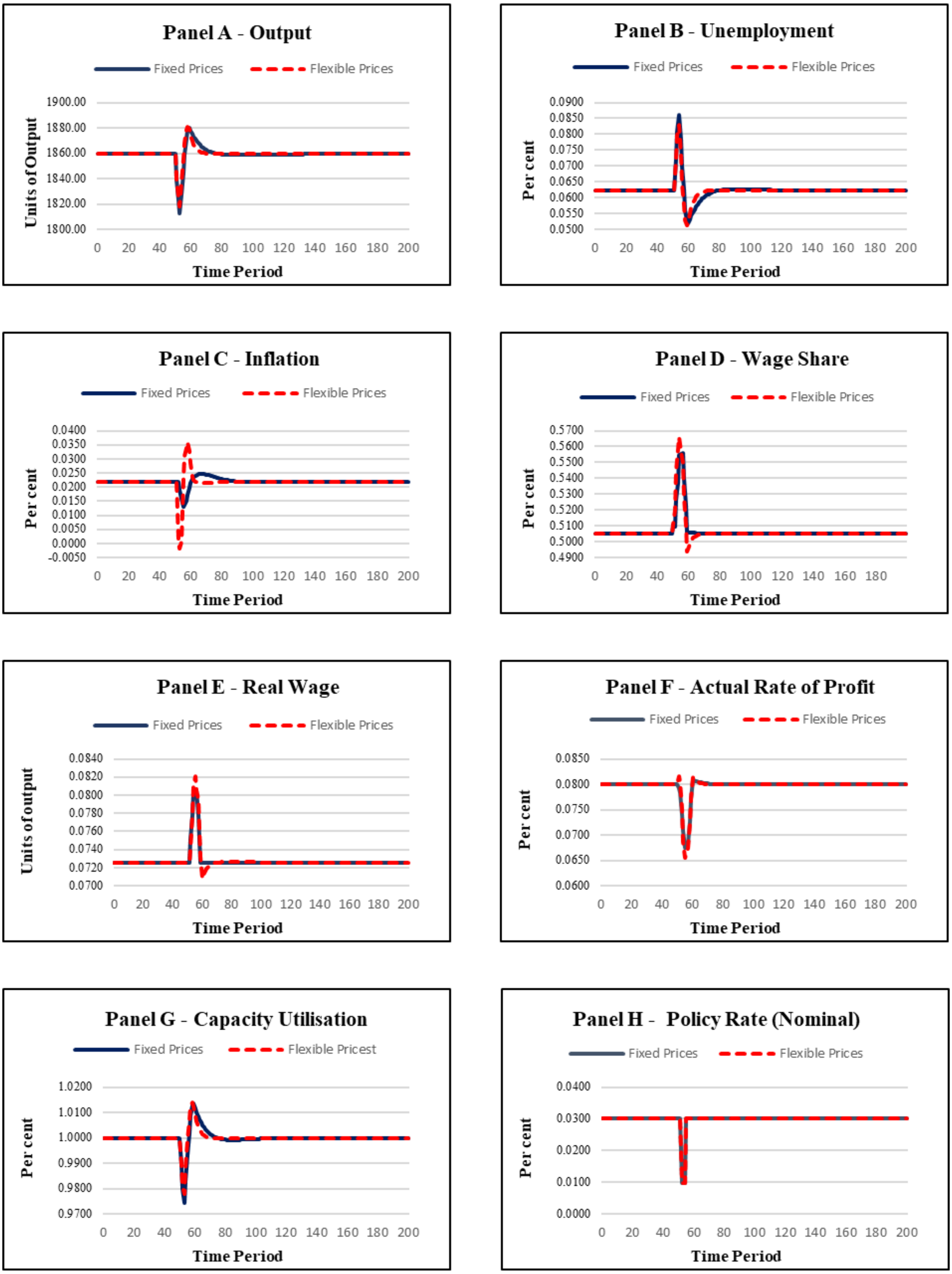

Figure 7: Time Paths of Key Variables for Negative Demand Shock (*Transitory*) and *Nominal Employment Target* for Monetary Policy - Fixed versus Flexible Prices.

inflation and capacity utilisation return to their long run time paths more quickly and with less volatility under the flexible price regime than under the fixed price regime. While there is, as before, no change to the real wage under the fixed price regime, real wages increase under the flexible price regime and facilitate the more rapid overall adjustment of the economy back to its original path because of the effects on aggregate demand of this change to distribution.

Figure 6 shows these adjustments under the *nominal employment target* approach to monetary policy The temporary policy rate change is shown clearly in Panel H when autonomous consumption initially falls in period 51 and then reverts to its original value in period 53. This policy rate reduction causes a fall in the target rate of profit and a jump in the real wage which is shown in Panel E. But the fall in autonomous consumption also causes a fall in the rate of capacity utilisation (seen in Panel G) so that under the flexible price regime, firms cut prices in response to this decrease in capacity utilisation. With fixed money wages, this increases the real wage. Real wages thus have two influences increasing them under the *employment target-flexible price* case: a reduction in the target rate of profit caused by the policy rate adjustment; and an additional boost because of the capacity utilisation effect on firm price-setting behaviour. Panel E indicates that the jump in real wages is thus slightly higher in the flexible price case than in the fixed price case. At the same time, however, it must be observed that the policy rate adjustment accounts for most of this change.

(b) Positive Cost Shock

The second type of shock to be considered is a 1% increase in the ambit claim of workers for increased money wages. This is represented by α in equation (24) of the structural model which was initially set at 2.4% in Table 1. Here we consider only a transitory version of this shock where α increases to 3.4% in period 51 for 4 quarters before returning to its initial value in period 55. We also consider this shock only for the case of the *employment target* approach to monetary policy which the above analysis suggests is the preferred approach to base level monetary policy. The effects of this second, baseline shock on the same set of key variables as considered for the demand shock are shown as the solid line in Panels A to H of Figure 8. What is immediately clear from this figure is the recursive nature of inflationary processes in the Sraffa-Keynes model. Only inflation is affected by the increase in α as is clear from Panel C. Inflation immediately begins to rise because of the increased cost associated with the higher demand for money wage increases by workers, but the overall increase in inflation across the four quarters of this "cost shock" is greater than the initial 1% increase in α as firms attempt to protect their profits, and a conflict emerges over income distribution (*cf*. Rowthorn 1977).

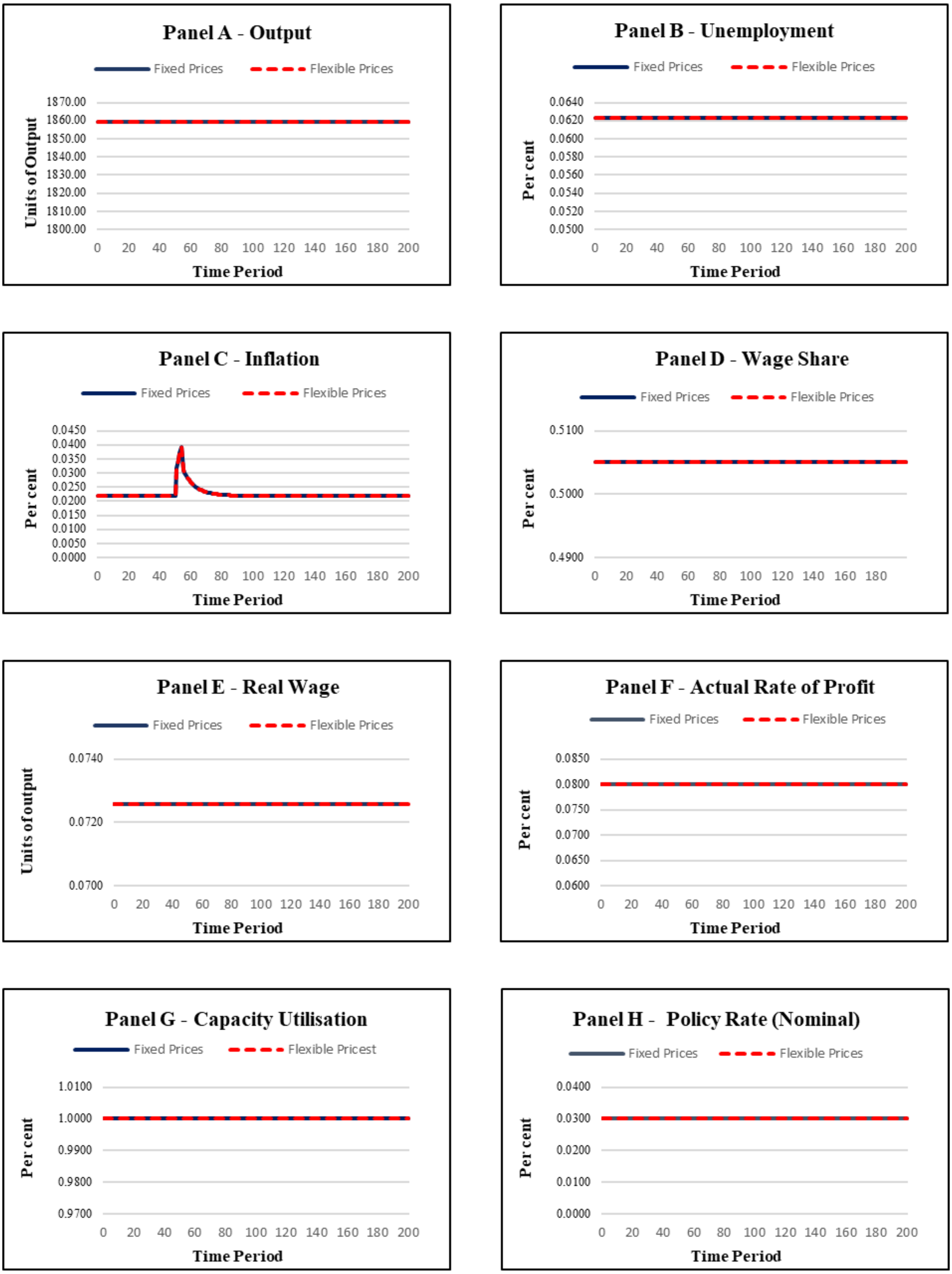

Figure 8: Time Paths of Key Variables for Positive Cost Shock (*Transitory*) and *Nominal Employment Target* for *M*onetary Policy - Fixed versus Flexible Prices.

Across the four quarters of the initial shock, inflation increases by about 2.4%, before beginning to decline as α returns to its original value after period 55. But no other variables in the model are affected by this shock. Since the policy rate is fixed to deliver an unemployment target, and the cost shock has no effect on the real economy, no change is necessary to the policy rate. Flow-on effects to income distribution are, therefore, absent. The same is true whether the fixed or flexible price regime is under consideration. In the end, the shock, being transitory, dissipates without any significant impact on the rest of the model.

9. Conclusion

This paper has sought to expand the suite of well-developed Post Keynesian models that can be used for the evaluation of monetary policy. The Post Keynesian literature outlines a variety of perspectives on the operation of monetary policy, the analysis of which requires the availability of such models (Wray 2007). The model developed in this paper falls within the Sraffa-Keynes tradition and is characterised by the principle of effective demand, Sraffa or target-return pricing which integrates the determination of key distributive variables and allows for short run cyclical price variations, conflict inflation, endogenous money, and a basic approach to monetary policy in the Smithin–Wray tradition of fixing the policy rate to achieve low or specified rates of unemployment. The paper argues that nominal interest rates are the appropriate target for monetary policy given the need to determine appropriate rates of return on capital and the good approximation that nominal rates provide for the particular specification that real rates take in the model.

A number of key results stand out from simulations of the model for two standard macroeconomic shocks. The first is that the full model with investment spending and prices responding to deviations of capacity utilisation from its desired level, is stable. The model returns to long period equilibrium whether standard macroeconomic shocks are temporary or persistent. The second is that this long period equilibrium is characterised by the achievement of the target rate of return, desired capacity utilisation, and Sraffian prices of production. The third is that monetary policy is transmitted through the typical Post Keynesian mechanism of variations in income distribution. Lower interest rates in response to a negative demand shock, for example, reduce the benchmark against which firms set their target rate of profit, though this occurs with a lag since the target rate of profit is based on the average of policy rates over several quarters. This has an impact on price setting, and this affects the real wage. These changes to distribution then affect aggregate spending, output and employment.

A fourth result is that flexible prices, where firms modify prices to cover the additional costs of running the capital stock at other than full capacity, has a similar effect on activity to monetary policy. Reductions in prices, for example, in response to lower rates of capacity utilisation after a negative demand shock, increase the real wage for a given money wage, and this causes a redistribution of income towards labour that in turn causes a stabilising increase in aggregate expenditure. Changes in monetary policy necessitated by a change in the structure of the model that modifies the employment target policy rate, simply reinforces this flexible price effect. Finally, the recursive nature of the model means that changes in costs affect only prices and have no real effects on the economy in the absence of any monetary policy response.

The paper thus provides some insight into the short run behaviour of Sraffa-Keynes models. While the long period is indeed characterised by the typical Sraffian relation between prices and income distribution, short run variations in aggregate demand have an effect on capacity utilisation which affects the actual rate of profit and thus income distribution, moderating the effect of the change in demand. In addition, if firms change their prices in response to variations in capacity utilisation (because this affects costs over the cycle) or if the central bank varies the policy rate in an attempt to maintain a given employment target, there are additional changes to income distribution that have a stabilising effect on aggregate demand. The same general relationships between prices and income distribution that operate in the long period therefore also operate in the short period, but the integration of the Sraffian price system with a Keynesian theory of output and employment means that variations in aggregate demand modify the outcome of these relationships in the short period. Sensible adjustment mechanisms, however, such as investment spending that responds to the deficient or excess capacity utilisation, ensure that such short run deviations of prices and distributive variables from their typical long period outcomes are corrected, and the system converges to a standard Sraffian characterisation in the long period.

The model outlined in this paper could be applied in a number of ways. It could, for example, be used to evaluate the various proposals for counter-cyclical monetary policy discussed in the Post Keynesian literature. Formal policy rules reflecting the various approaches to counter-cyclical monetary policy described in the literature could replace the *constant nominal rate* or *nominal employment target* approaches simulated in this paper, and the time paths of key variables compared to evaluate which approach delivers the best outcomes. Something like this could also be done to compare monetary policy and fiscal policy responses to a range of macroeconomic shocks.

The model could also be improved in a number of ways. It would be highly desirable to expand the number of sectors to explore the impact that inter-sectoral linkages have on the outcomes reported. Such a model would provide richer insights into the impact of monetary or fiscal policy responses to various shocks on the *structure* of the economy as well as on its overall performance. The short period-long period nexus could also be given further attention by allowing the growth rate of autonomous spending to be positive, and examining whether the various monetary policy responses alter the long period growth path in addition to their effect of the economy's cyclical fluctuations. Work also needs to be done on comparing the impact of various shocks and policy responses to these shocks across a range of models to ascertain common and unique insights that such models might provide on the effects of these events.

References

Aspromourgos T. (1991), "Inflation in a Sraffa-Keynes Framework", *Economies et Societes*, 8, November-December, pp.107-126.

Aspromourgos T. (2004), "Sraffian Research Programmes and Unorthodox Economics", *Review of Political Economy*, 16 (2), pp.179-206.

Aspromourgos T. (2011), "Can (And Should) Monetary Policy Pursue a Zero Real Interest Rate, Permanently?", *Metroeconomica*, 62 (4), pp.635-655.

Cesaratto S., Serrano F. and Stirati A. (2003), "Technical Change, Effective Demand and Employment", *Review of Political Economy*, 15 (1), pp.19-32.

Commonwealth of Australia (2018), *Budget 2018-19: Budget Strategy and Outlook, Budget Paper No. 1*, Canberra: Australian Government.

Cottrell A. (1994), "Post-Keynesian Monetary Economics", *Cambridge Journal of Economics*, 18, pp.587-605.

Docherty P. (2012), "Long Period Interest Rate Rules in a Demand-led Kaldor-Pasinetti-Sraffa-Keynes Growth Model, *Journal of Post Keynesian Economics*, 34 (3), pp.521-546.

Dow S. (2013), "Teaching Open-System Economics", in Jespersen J. and Madsen M.O. (eds.), *Teaching Post Keynesian Economics*, Cheltenham UK and Northampton, MA: Edward Elgar, pp.73-87.

Dumenil G. and Levy D. (1999), "Being Keynesian in the Short Term and Classical in the Long Term: The Traverse to Classical Long-Period Equilibrium", *The Manchester School*, 67 (6), pp.684-716.

Eatwell J. and Milgate M. (1983), "Introduction", in Eatwell J. and Milgate J. (eds.), *Keynes's Economics and the Theory of Value and Distribution*, London: Duckworth Press, pp.1-17.

Fontana G. and Palacio-Vera A. (2006), "Is There an Active Role for Monetary Policy in the Endogenous Money Approach?", in Gnos C. and Rochon L.P. (eds.), *Post Keynesian Principles of Economic Policy*, Cheltenham, UK and Northampton, MA, USA: Edward Elgar, pp.49-56.

Freitas F. and Serrano F. (2015), "Growth Rate and Level Effects, the Stability of Adjustment of Capacity to Demand and the Sraffian Supermultiplier", *Review of Political Economy*, 27

(3), pp. 258-281.

Friedman M. (1968), “The Role of Monetary Policy”, *American Economic Review*, 58 (1), pp.1-17.

Garegnani P. (1992), “Some Notes for an Analysis of Accumulation”, in Halevi J., Laibman D. and Nell E.J. (eds.), *Beyond the Steady State: A Revival of Growth Theory*, London: Macmillan, pp.47-71.

Goodfriend M. (2007), “How the World Achieved Consensus on Monetary Policy”, *Journal of Economic Perspectives*, 21 (4), pp.47-68.

Gordon R.J. (2011), “The History of the Phillips Curve: Consensus and Bifurcation”, *Economica*, 78, pp.10-50.

Kaldor N. (1955-56), “Alternative Theories of Distribution”, *Review of Economic Studies*, 23 (2), pp.83-100.

Kaldor N. (1986), *The Scourge of Monetarism*, 2nd edn, Oxford: Oxford University Press.

King J.E. (2015), *Advanced Introduction to Post Keynesian Economics*, Cheltenham UK and Northampton, MA: Edward Elgar.

Kurz H.D. and Salvadori N. (1995), *Theory of Production: A Long-Period Analysis*, Cambridge: Cambridge University Press.

Lavoie M. (2003), “Kaleckian Effective Demand and Sraffian Normal Prices: Towards a Reconciliation”, *Review of Political Economy*, 15 (1), pp.53-74.

Lavoie M. (2006), “Do Heterodox Theories Have Anything in Common? A Post-Keynesian Point of View”, *Intervention Journal of Economics*, 3 (1), pp.87-112.

Lavoie M. and Seccareccia M. (1999), “Interest Rate: Fair”, in O’Hara P. (ed.), *Encyclopedia of Political Economy, Volume 1*, London: Routledge, pp. 543-545.

Lee F.S. (1994), “From Post Keynesian to Historical Price Theory, Part 1: Facts, Theory and an Empirically Grounded Pricing Model”, *Review of Political Economy*, 6 (3), pp.303-336.

Milgate M. (1982), *Capital and Employment: A Study of Keynes’s Economics*, London: Academic Press.

Moore B.J. (1988), *Horizontalists and Verticalists: The Macroeconomics of Credit Money*, Cambridge: Cambridge University Press.

Moore B.J. (1994), “The Demise of the Keynesian Multiplier: A Reply to Cottrell”, *Journal of Post Keynesian Economics*, 17 (1), pp.121-133.

Palley T.I. (2007), “Macroeconomics and Monetary Policy: Competing Theoretical Frameworks”, *Journal of Post Keynesian Economics*, 30 (1), pp.61-78.

Panico C. (1988), *Interest and Profit in the Theories of Value and Distribution*, London: Macmillan.

Pasinetti L.L. (1974), *Income Distribution and Growth*, Cambridge: Cambridge University Press.

Pasinetti L.L. (1977), *Lectures on the Theory of Production*, London: Macmillan.

Pivetti M. (1985), “On the Monetary Explanation of Distribution”, *Political Economy: Studies in the Surplus Approach*, 1 (2), pp.73-103.

Rochon L.P. and Setterfield M. (2008), “The Political Economy of Interest Rate Setting, Inflation, and Income Distribution”, *International Journal of Political Economy*, 37 (2), pp.5-25.

Rowthorn R.E. (1977), “Conflict, Inflation and Money”, *Cambridge Journal of Economics*, 1, pp.215-239.

Rochon L.P. (2007), "The State of Interest Rate Policy: Where are We and Where are We Going?", *Journal of Post Keynesian Economics*, 30 (1), pp.3-11.

Rochon L.P. and Setterfield M. (2007), "Interest Rates, Income Distribution, and Monetary Policy Dominance: Post Keynesians and the 'Fair Rate' of Interest", *Journal of Post Keynesian Economics*, 30 (1), pp.13-42.

Rochon L.P. and Setterfield M. (2008), "The Political Economy of Interest Rate Setting, Inflation and Income Distribution", *International Journal of Political Economy*, 37 (2), pp.5-25.

Sanfilippo E. (2011), "The Short Period and the Long Period in Macroeconomics: An Awkward Distinction", *Review of Political Economy*, 23 (3), pp.371-388.

Serrano F. (1995), "Long Period Effective Demand and the Sraffian Supermultiplier", *Contributions to Political Economy*, 14, pp. 67-90.

Smithin J. (2007), "A Real Interest Rate Rule for Monetary Policy?", *Journal of Post Keynesian Economics*, 30 (1), pp.101-118.

Sraffa P. (1960), *Production of Commodities by Means of Commodities*: *Prelude to a Critique of Economic Theory*, Cambridge: Cambridge University Press.

White G. (1996), "Capacity Utilisation, Investment and Normal Prices: Some Issues in the Sraffa-Keynes Synthesis", *Metroeconomica*, 47 (3), pp.281-304.

White G. (1998), **"**Disequilibrium Pricing and the Sraffa-Keynes Synthesis", *Review of Political Economy*, 10 (4), pp.459-75.

Wray L.R. (2007), "The Political Economy of Interest Rate Setting, Inflation and Income Distribution", *Journal of Post Keynesian Economics*, 30 (1), pp.119-141.

Appendix

(i) Real versus Nominal Profit Rate

To see that the nominal rate of profit may be a good approximation for the real rate in our analysis when p_{t-1} is used in equation (8) in the text to define the rate of profit, consider the following. Replace p_t in equation (8) with p_{t-1} and rearrange to obtain the following:

$$r_{n,t} \cdot p_{t-1} \cdot K_t = p_t \cdot Y_t - M_t \cdot N_t - p_t \cdot \delta \cdot \nu \cdot Y_t \tag{A1}$$

Divide (A1) by $p_t Y_t$, to obtain:

$$r_{n,t} \cdot \nu \cdot p_{t-1}/p_t = 1 - w_t \cdot \ell_t - \delta \cdot \nu \tag{A2}$$

where everything has the same meaning as in the text but we retain the term p_{t-1}/p_t on the left hand side of the expression. Since, by definition, $p_t = p_{t-1}(1 + \pi_t)$, we may express the ratio p_{t-1}/p_t as $1/(1 + \pi_t)$. In this case, (A2) becomes:

$$\nu \cdot [r_{n,t}/(1 + \pi_t)] = 1 - w_t \cdot \ell_t - \delta \cdot \nu \tag{A3}$$

If we define $r_{n,t}/(1 + \pi_t)$ as a kind of *real* rate of profit, since it adjusts the nominal rate of profit for inflation, and if we designate this as $r_{r,t}$, our wage-profit relation becomes:

$$w_t = \frac{1}{\ell_t} \cdot [1 - \nu \cdot (r_{r,t} + \delta)] \quad \text{(A4)}$$

Using $r_{n,t}/(1+\pi_t)$ for $r_{r,t}$ in this equation is likely to generate non-linearities when the overall model's reduced form is obtained and it will be convenient at this point to approximate this term. The standard candidate for approximation is the usual one for the real rate of return, where $r_{r,t} = r_{n,t} - \pi_t$. This is, however, a poor approximation. For the case where $r_{n,t}$ is in the neighbourhood, for example, of 0.08 and π_t is of the order of 0.03, $r_{n,t}/(1+\pi_t)$ would be 0.0777. Approximating this with $r_{n,t} - \pi_t$ would yield 0.05 and would generate an approximation error of 0.0277. Simply using the nominal rate itself as an approximation would involve an approximation error of only 0.0023. Similar results obtain for a wide range of plausible values for $r_{n,t}$ and π_t.

(ii) Solution and Stability of Equilibrium Income

Equation (43) in the text is a second order linear difference equation in Y_t. Its solution is relatively straightforward. The following potential solution to the particular integral of (43): $Y_t = Y_{t+1} = Y^*$, yields:

$$Y^* - \theta_1 \cdot Y^* = \bar{A}$$

For which the solution is the familiar expression:

$$Y^* = \frac{1}{1-\theta_1} \cdot \bar{A} \quad \text{(A5)}$$

where, as in the text: $\lambda_1 = i^* + \sigma_k + \delta$;

$\lambda_2 = [1 - \nu \cdot \lambda_1] \cdot (1-\tau) \cdot (s_c - s_w)$;

$\lambda_3 = (1 - s_c) \cdot (1-\tau) \cdot (1 - \delta \cdot \nu)$; and

$\theta_1 = \lambda_2 + \lambda_3 + \delta \cdot \nu$.

The stability of this equilibrium depends on the solution to the following complementary function:

$$Y_{t+1} - \theta_1 \cdot Y_t = 0$$

Utilising the following potential solutions to this equation: $Y_t = Bb^t$ gives:

$$Bb^{t+1} - \theta_1 \cdot Bb^t = 0$$

$$Bb^t \cdot (b - \theta_1) = 0$$

A non-trivial solution requires the bracketed term to equal zero which implies the following eigenvalue for the output equation:

$$b = \theta_1 \tag{A6}$$

The equilibrium value for output in (A5) will thus be stable if:

$$\theta_1 < 1 \tag{A7}$$

which is precisely the condition under which the output multiplier will exceed unity.

(iii) Solution and Stability of Equilibrium Inflation

The equilibrium value for expected inflation, π_t^e, depends on the particular integral of (41). Using the following potential solution $\pi_t^e = \pi_{t+1}^e = \pi_t^{e*}$, yields:

$$\pi_t^{e*} - [1 - h_3(1 - h_1)] \cdot \pi_t^{e*} = h_3 \cdot (\alpha - h_2 \cdot UE^*)$$

For which the solution is:

$$\pi_t^{e*} = \frac{\alpha - h_2 \cdot UE^*}{1 - h_1} \tag{A8}$$

Once again, the stability properties of this equilibrium depend on the solution to the complementary function:

$$\pi_{t+1}^e - [1 - h_3(1 - h_1)] \cdot \pi_t^e = 0$$

Utilising the following potential solution: $\pi_t^e = Dd^t$, gives:

$$Dd^{t+1} - [1 - h_3(1 - h_1)] \cdot Dd^t = 0$$

$$Dd^t \cdot \{d - [1 - h_3(1 - h_1)]\} = 0$$

A non-trivial solution requires:

$$d - [1 - h_3(1 - h_1)] = 0$$

which implies:

$$d = [1 - h_3(1 - h_1)] \tag{A9}$$

The equilibrium value for expected inflation in (A8) will thus be stable if:

$$[1 - h_3(1 - h_1)] < 0$$

or

$$h_1 < 1 \tag{A10}$$

(iv) Determination of the Nominal Employment Target Policy Rate

The nominal employment target approach to monetary policy sets the policy rate in order to achieve a particular unemployment rate outcome. Let this unemployment target be given by UE^T. We may rearrange equation (48) in the text to express the level of output which delivers this unemployment target be identified as Y^T.

$$Y^T = (\bar{N}/\ell) \cdot (1 - UE^T) \quad \text{(A11)}$$

Substituting this value into the expression for equilibrium income (A5) above we obtain:

$$(\bar{N}/\ell) \cdot (1 - UE^T) = \frac{1}{1 - [\lambda_2 + \lambda_3 + \delta\nu]} \cdot \bar{A} \quad \text{(A12)}$$

Since it is the λ_2 term that contains the policy rate, we rearrange this expression to obtain this term explicitly:

$$\lambda_2 = 1 - \lambda_3 - \delta\nu - \frac{\bar{A}}{(\bar{N}/\ell) \cdot (1 - UE^T)} \quad \text{(A13)}$$

Letting:

$$\lambda_4 = 1 - \lambda_3 - \delta\nu - \frac{\bar{A}}{(\bar{N}/\ell) \cdot (1 - UE^T)} \quad \text{(A14)}$$

And substituting for λ_2 gives:

$$[1 - \nu \cdot (i^* + \sigma_k + \delta)] \cdot (1 - \tau) \cdot (s_c - s_w) = \lambda_4 \quad \text{(A15)}$$

Then rearranging to obtain i^* gives:

$$i^* = \frac{1}{\nu} \cdot [1 - \frac{\lambda_4}{(1 - \tau) \cdot (s_c - s_w)}] - (\sigma_k + \delta) \quad \text{(A16)}$$

Which gives the policy rate that will deliver the desired unemployment rate.